Technology Competition on Social Networks - Paradoxes and Resolution

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Abstract

Competition of incompatible technologies can lead to outcomes where more than one technology takes a significant share of the market. Instant messengers are one such example, since a significant fraction of people uses more than one messenger, and no provider dominates the market. This outcome is contrary to the usual prediction of the technology adoption literature. To resolve this paradox, an agent-based model of instant messenger adoption is used to study the effect the social network structure has on the diffusion of this technology. A mathematical model is then used to predict certain outcomes and allow further analysis of the dynamics of adoption.

1 Introduction

Adoption and diffusion of incompatible technologies has been a topic of great interest in economics [9][8][5][11]. Examples of these scenarios include 3.5” vs. 5 1/4” floppy drives, the Mini Disc vs. the DAT, and currently Blue-Ray vs. HD-DVD [13]. Three

common features associated with the competition of these types of technologies are path-dependence, irreversibility, and potential regret [6]. Path-dependence refers to the fact that a very small initial difference in market shares can have a significant effect in the long-run, due to the strong network externalities that play a role in technology competition. The benefit of owning a fax machine is zero if nobody else has one, but as more people adopt a fax machine, the benefit increases, since the possibility of faxing more people makes the good more useful. This creates positive feedback loops that can transform small differences in market shares to a large gap in a short amount of time, often leading to outcomes where one technology dominates the market completely. One prominent example of this is the VHS system competing with the Betamax system in the eighties, where VHS emerged as a winner, and Betamax disappeared from the market. For some observers this was an inferior outcome because Betamax was seen by some experts as the technologically superior system. However, the outcome was virtually irreversible, since there was no incentive to buy a Betamax anymore when most people had a VHS, since the benefit of having either one is an increasing function of the number of people who already have it, due to an increased amount of videos available for that system, the ability to exchange movies with other people, and a better understanding of how the system functions. This is the situation where potential regret comes into play, since inferior outcomes can emerge through chance events that put one technology in front of others early in the game. This is especially true in the early stages of technology adoption, when experience with different products is very limited, and hence judgments as to which one is the best can not be made easily. In most cases studied there is also a substantial fixed cost associated with the adoption of one technology, and there is mostly only use for one product. For instance, the typical consumer would neither find it useful nor within her budget to have both a VHS and a Betamax system, since both solve the same problem she faces, namely playing and recording videos.
Instant messengers are computer applications that allow two or more people who own the program to communicate in an instantaneous way, mostly by means of text messages (although with recent advances in Voice over IP, audio conversations have been integrated into major messengers as well). Each person on a messenger network has an identity, such as a nickname, or a serial number, in order to log on to the system, and to be able to add other people to a list of friends. The messenger will then show who of these friends are on the system concurrently, and allow for text, audio, and video conversations as well as file or link exchanges. The major messenger networks are free of charge, but contain links to advertisements or the networks' web portals (such as Yahoo!, MSN, and AOL). The major networks, AIM (meaning both the proprietary AOL messenger and the freely available one), MSN Messenger, and Yahoo Messenger, are all incompatible, meaning that people having an AIM account

are not able to talk to people who have an MSN account[1] Standard theory would suggest that one messenger would take control of the entire market, given the strong network effects at work. However, many users maintain more than one messenger [1] [2], in order to talk to friends in multiple networks. Furthermore, even though AOL had a leading share of the messenger market in the late nineties, MSN and Yahoo! have captured a large market share despite their late entrance. This poses interesting questions about what kind of behavior on behalf of the consumers and what kind of network characteristics make this kind of scenario a relatively stable one. Another interesting economic question that will not be discussed in this paper is why instant messenger providers immediately released their products without a fee.[2]

Contrary to the competition between Betamax and VHS, the situation with instant messengers is a bit different, in that if a user's friends have different messengers, it might be beneficial to the user to install more than one messenger, in order to talk to each one of his friends. This requirement arises from the incompatibility of different messengers, since a person who has AIM installed is not able to talk to a person who has MSN Messenger installed. Also, since messengers are free to download, the consumer will only incur non-financial costs, allowing for multiple messengers to be installed without any budget considerations. The network externalities of messengers are also much more local in nature, since many people use messengers to exchange messages with friends from their social network, so they will be much more interested in which messengers their friends use than people they do not know. In contrast to this, there are much more global network effects in the VHS and Betamax case , since - for instance - the supply of rental movies in either format depends on the market shares of the systems. As soon as one of the systems has a significant lead on the other, rental videos will appear predominantly in the leading format, since suppliers want to cater to the largest fraction of the population possible. This creates another positive feedback loop in the supply side of the market, tipping the market further towards a winner-take-all scenario. Part of this difference arises from messengers being a stand-alone technology, not requiring any additional component - other than people to talk to - to be operated. The complementarities here arise not between two products from the supply side, but rather from the messengers people use and the connections in the social network, so that the additional feedback from

[1]It should be noted that recently Yahoo! and MSN have joined forces in their messenger network, by working on making their systems interoperable, partly as a response of newly emerging messenger GoogleTalk working with AOL to make their systems interoperable as well [4] .

[2]Although AOL started out only offering the capability of sending messages within its own subscriber network, as soon as ICQ released a free version of its messenger, AOL released a freely available messenger as well [3].

the supply side that would affects everyone in the market does not take place here [3]. Only if the connectivity in the network is sufficiently high should a user's adoption decisions have global consequences. The spread of messengers in a social network and resulting market shares should therefore be influenced by the topology of the social network, and not only by the initial market shares of a messenger.

2 Motivation

The main goal of this paper is to investigate the effect that changing the network structure has on the distribution of messengers in the social network. We will investigate under which circumstances more than one messenger takes hold in the market, and which parameter regions prohibit this from happening. The usage patterns, and their dependence on parameters, will also be subject to investigation. When more than one messenger is present in the market, characteristics of the nodes that have more than one messenger installed will be studied. Finally, the structure of the resulting IM networks and their correlation with the underlying social network structure will be discussed.

There are many reasons why it is important to determine what exactly is responsible for the anomalous (compared to similar scenarios) situation in the messenger market. First, this is a situation where the structure network is extremely important in determining the outcome, since many decisions are made locally. Normally, network effects are assumed to occur on a global level, i.e., everyone's decision is linked to everyone else's decision. From this assumption, rules of thumb like Metcalf's law[4] have been devised, and presumably been used by practitioners and policymakers in their decisions. However, these beliefs are not valid for this type of network good, where benefits are mostly locally derived from the social network a person lives in. Products where this kind of interaction is the norm are increasingly appearing on the market, as companies are discovering the value of advertising on social networking sites such as MySpace, ClassMates, Facebook, and others. There should therefore be theoretical work allowing for local network effects, by making the network explicit in the model. The usual network good case could then still be modeled by making

[3] Certain information goods might be complementary to messengers. For instance, if someone uses Yahoo! Mail, it might yield additional benefits if this is used together with Yahoo! Messenger. This is especially true in the case of Gmail, Google's e-mail service. Here, logging onto the account in a browser allows one to use Google Talk without actually installing the stand-alone application on the computer.

[4] Metcalf's law states that the benefit of a network good scales as the square of the number of people using that good, which can be derived from the fact that the benefit is increasing in the number of people who use the good, and each person is connected to everyone else.

the network such that everyone is connected to everyone else.
Second, messengers are just an analogy for many other goods where local interactions play a large role. Any good which has negligible adoption cost and where the benefit depends on the immediate neighborhood in some social network should have a similar dynamic. One such example is perhaps programming languages. One can adopt a new language easily, since the information to do so is freely available. It is also beneficial for a team of programmers to know the same language, since then code can be exchanged, combined, and modified more easily than if everyone was using a different language. Insights generated for messengers could carry over to these types of goods in general.
What the paper does not intend to explain is why people initially choose to install messengers, or which groups constitute the initial adopters. Everyone in the network will be given equal probability of having a any messenger. Variations of this rule are possible, and interesting, but the emphasis here is on the spread of instant messenger from an initial seed. This model also ignores any strategic interactions between friends in the network, and concentrates instead on individual decision-making based on local, myopic information about the network and the future consequences of one's actions. This should by no means imply that people do not strategically interact in these decisions, since there are elements of coordination in messenger choice that might deserve further attention, perhaps in a more game theoretic framework. Another strategic element that has been discussed by Lee[10] is the timing of entrance into a market where the network structure is explicitly modeled. This question will also not be dealt with in this paper.

2.1 Costs of a messenger

Messengers occupy some space on the user's hard drive and also use some working memory while running. Since they are an online application, they will also use some of the bandwidth of the user's internet connection. Given that messengers are not a very demanding application in terms of memory and bandwidth, the more salient cost is the cost of learning to use the messenger and the cost of maintaining it (adding new contacts, deleting or blocking existing contacts, customizing the messenger).
In the model, the cost structure is simplified substantially. The cost of adopting the messenger, C_a will represent the reluctance of the user to install a new messenger, which could reflect the time and effort cost of downloading, installing, and configuring it. It is reasonable to assume that there is also some cost of running a messenger once it has been installed, reflecting the actual hardware requirements and a psychological cost associated with the nuisance of having an additional pro-

gram running. The cost of the messenger running on the system is denoted by C_d, where the d stands for deletion, since if the benefit of the messenger is less than or equal to C_d, the user will delete the messenger.

2.2 Benefits of a messenger

The main benefit of a messenger is measured by the number of people that it enables one to communicate to, weighted by their respective importance. In the model, we will abstract away any heterogeneity in the strength of social contacts. Also, what really matters is not the gross number of people the user can talk to through a messenger, but instead the number of people who can only be contacted through one messenger. This means that if a person can talk to another person through two different messengers, then this contact will not be counted towards the benefit of any one of these messengers, since deleting one of them would not mean any loss.

2.3 Assumptions about agents

We assume the agents to be rational in the sense that their goal is to maximize the number of friends they can talk to while minimizing the number of messengers, subject to the constraint that only one messenger can be adopted or deleted every time an agent is updated. This is an important assumption, since one could explain why people use multiple messenger by simply assuming that as soon as people install a messenger, they never take the time to delete it again, or that people do not pay attention to overlaps. This, however, would discount the effect the network structure has on the system. Our goal is to look at how agents with simple adoption rules that to a certain extent capture the benefits and costs of messengers react to different network structures. Even though the decision model might not be the most realistic, it is a good starting point to see how network structure influences local decisions and global outcomes.

3 Computational Model

3.1 Social Structure

The model consist of a fixed number n of agents from an agent set $N = \{1, 2, ..., n\}$ that are connected in some way to each other. These connections can be represented by a graph, where the vertices are the agents, and the edges are connections between agents. With n agents, the social network structure can be represented by a matrix F, standing for friendship, defined by

$$F_{ij} = \begin{cases} a, \text{if j is connected to i} \\ 0, \text{otherwise} \end{cases}$$

This is the adjacency matrix of the graph. We assume that if i is connected to j then j is also connected to i, which in most cases is a reasonable assumption to make. In any event, the assumption is justified by giving a connection between i and j the interpretation that i wants to communicate with j, and that this intention is reciprocated. This means that the resulting matrix will be symmetric. Another rather trivial assumption about the adjacency matrix is that for each i, $F_{ii} = 0$, since one agent cannot communicate with itself. The last assumption is that agents either want to communicate with each other, or not, so that $a = 1$. In general, this must not be the case, because in real social networks the strength of the intention to communicate with each other is not the same for all friends. The network is also assumed to be static, meaning that no new connections are formed. The network will be exogenously determined and static throughout any given simulation, since the focus of this paper is not to explain why people want to communicate with each other via IM in the first place. Three types of graphs will used for the social network: A random graph with a binomial degree distribution, a random graph with a scale-free degree distribution, and a "Small-Worlds" graph as proposed in[12].
The random graph with a binomial degree distribution is generated as follows:

1. Start with an n by n zero matrix, F. Fix a probability p.

2. For each i and each $j \neq i$, let $F_{ij} = F_{ji} = 1$ with probability p

The resulting mean degree (number of neighbors) $\bar{k}$ will be approximately np. This parameter will be changed in the simulations.
The random graph with scale-free degree distribution is generated slightly differently[5]:

1. Start with an n by n zero matrix, F. Fix α[6].

2. Generate a degree distribution $\{k_1, k_2, \cdots, k_n\}$ according to $P(k_i = k) = \beta k^{-\alpha}$, where β is a normalizing constant [7].

[5] The degree distribution induced is not truly a power law distribution, since the maximum degree is given by $n-1$, while there is no upper bound on the degree in a true power-law distribution. Effectively, the distribution generated will assign $P(k_i \geq n-1) = P(k_i = n-1)$. However, asymptotically this will be a power-law distribution.

[6] The parameter α determines how fast the power law distribution falls off.

[7] β can be calculated from the requirement that $\sum_{k=0}^{n-1} \beta k^{-\alpha} = 1$.

3. If $\sum_{i=1}^{n} k_i$ is odd, repeat step 2.
4. For each i, connect node i to k_i other nodes uniformly at random
5. Update F accordingly.

Finally, the Small-Worlds graph is created as follows[8]:

1. Arrange the nodes on a ring and connect each node with its nearest and next-nearest neighbors, which yields a regular graph of degree 4.
2. Next, give every pair of nodes that is not yet connected probability γ of connection.
3. Create F according to the connections in the graph

This graph introduces clusters into the social network, since there will be parts of the graph that are very connected and parts that only have few connections between each other, due to the initial lattice structure that remains unchanged.

3.2 Technological Structure

There is an ordered set $M = \{A, B, ...\}$ of messengers to choose from. The cardinality of M will be denoted by m. The cases studied will be restricted to m being 1, 2, and 5. Messengers will be referred to by upper case letters, and their index in M by the same lower case letter. Any agent can have any combination of these messengers or no messengers at all. Initially, all agents have no preference for any particular messenger, so that each agent is completely defined by his position in the network and the messengers he owns. Agent i's state is defined by the set of messengers he has installed and denoted by $S \subset 2^M$. It immediately follows that there are 2^m possible states each agent can be in. We define the technology ownership matrix T by setting T_{kl} to 1 whenever agent l owns the kth technology in M, and to 0 otherwise. Agent i and agent j are said to be connected through messenger K, if $F_{ij} = 1$ and $K \in S_i \cap S_j$, meaning that they are connected in the friendship network and both have technology K. This relation induces a connection network N^k for each technology K, which is a subset of the underlying social network F. Its entries are defined by

$$N_{ij}^k = F_{ij} \cdot T_{ki} \cdot T_{kj},$$

meaning that if N_{ij}^k is non-zero, agent i and j are connected through technology K.

[8]The procedure is slightly different from Strogatz/Watts in that the initial lattice structure stays intact. In their procedure, existing links from the initial configuration can be destroyed.

3.3 Dynamics and Decision Algorithm

Time is discrete in the model, representing the decisions of individual agents. The number of periods in each simulation is defined by $T = 25n$, so that in each run of a simulation the expected number of updates per agent is 25[9]. Each time period a randomly selected agent will consider either adopting a new messenger or deleting an installed messenger with equal probability. If agent i considers adopting, the following algorithm is used to make a decision:

1. Put all messengers that are currently not installed into a set A_i. If all messengers are installed, stay in current state.
2. For each messenger in A_i, determine $\mathrm{MB}_i(K)$, the marginal benefit of adoption, defined as the number of additional people that the agent would be connected to if he installed the messenger in question.
3. Let $\hat{A}_i$ be the subset of A_i containing all messengers that have marginal benefit greater than or equal to C_a, the cost of adoption.
4. If $\hat{A}_i$ is empty, stay in current state. Otherwise, choose the messenger with the highest marginal benefit among the remaining messengers in $\hat{A}_i$ and install it. If there are ties, choose on at random among the tied to install. Call the choice made a_i.
5. Update T and N^K.

More formally, we can define the following quantities which allow calculation of agent i's choice:

$$
\begin{aligned}
A_i &= \{K \in M | T_{ki} = 0\}, && (1)\\
\mathrm{MB}_i(K) &= \sum_{j \neq i} F_{ij} T_{kj} \prod_{l \neq k} (1 - N^l_{ij}) && (2)\\
\hat{A}_i &= \{K \in A_i | \mathrm{MB}_i(K) \geq C_a\} && (3)\\
a_i &= \operatorname*{argmax}_{K \in \hat{A}_i} \mathrm{MB}_i(K) && (4)
\end{aligned}
$$

It should be noted that if there is more than one messenger that maximizes $\mathrm{MB}_i(K)$, then formally a_i will be a set. It should then be understood that a random element of

[9] Test runs have indicated that this is about the time it takes for the system to settle into a steady state.

this set is chosen, and referred to as a_i, now being a messenger. The same comment holds for the deleting algorithm below.

On the other hand, if agent i considers deleting a messenger, the following similar algorithm is used to determine his choice:

1. Put all messengers that are currently installed into a set D_i. If no messenger is installed, stay in current state.

2. For each messenger in D_i, determine MC_i, the marginal cost of deleting, defined as the number of people who the agent would not be connected to anymore through any other messenger if this messenger were deleted.

3. Let $\hat{D}_i$ be the subset of D_i containing all messengers that have marginal cost less than or equal to C_d, the reluctance to delete a messenger when it is installed.

4. If $\hat{D}_i$ is empty, stay in current state. Otherwise, choose the messenger with the lowest marginal cost among the remaining messengers in $\hat{D}_i$ and delete it. If there is more than one that fits this criterion, choose on at random and delete it. Call the choice made d_i.

5. Update T and N^K.

Again, this can be expressed more formally as:

$$D_i = \{K \in M | T_{ki} = 1\}, \tag{5}$$

$$\mathrm{MC}_i(K) = \sum_{j \neq i} T_{kj} \prod_{l \neq k} (N_{ij}^k - N_{ij}^l) \tag{6}$$

$$\hat{D}_i = \{K \in D_i | \mathrm{MC}_i(K) \leq C_d\} \tag{7}$$

$$c_i = \operatorname*{argmin}_{K \in \hat{D}_i} \mathrm{MC}_i(K) \tag{8}$$

These decision rules capture some of the tradeoffs inherent in choosing instant messengers. The possibility of overlaps between different messengers is accounted for by the agent in making a decision. Given information on what type of messengers her immediate neighbors are using, the user is able to maximize the number of people she can talk to while minimizing the amount of messengers necessary to do that, with the restriction that only one messenger can be deleted or installed every time she is updated. More sophisticated rules could be devised that allow for more than one messenger to be deleted or installed at once.

3.4 Parameters of the model

Since the decision algorithm will stay fixed throughout all simulations, the only two parameters that will have a direct effect on the decisions are C_a and C_d. To make the presentation more concise, these will be referred to by a vector $C = (C_a, C_d)$. As multiple test runs have indicated, interesting behavior is mostly seen when $C \in \{(3,1),(4,1),(5,2),(6,3)\}$, so most discussion will be restricted to these values. The more important parameters are the ones controlling the social structure. The most obvious of these is n, the number of agents in the social network. In the random graph with binomial degree distribution, the most important parameter is p, the probability of any two given nodes being connected. This has a direct impact on the mean degree of the graph, meaning the mean number of people an agent is connected to, which is given by $(n-1)p$, since there are n agents in the network, and there are no self-edges allowed. p will be varied so that the mean degree is between 5 and 20. In the scale-free random graph case, the degree distribution is given by $P(k) = \beta k^{-\alpha}$, where α is an exponent determining how fast k, the degree, falls off, and β is a normalizing constant. This graph will be used to investigate which effect changing the degree distribution, while leaving the randomness of connections intact, has on different outcome variables.
The third type of graph is created by a procedure described in [12], where initially a regular graph, one where each agent has the same degree, is created, and then each unconnected pair of agents is wired to each other with probability γ. When β is zero, the transitivity of the graph is very high. This means that if agent i is agent j's neighbor, and agent j is agent k's neighbor, then there is a high probability that agent i is agent k's neighbor. This is a characteristic of many social networks, and hence an important feature to include in the social structure. However, if γ is zero, the mean shortest path length between any two vertices will be larger than in a real-life network, where path-lengths have generally been found to be relatively short. Therefore, γ will be varied in this model to see what effect a decreasing mean path length has, when clustering is high.
In addition to the graph type, and various parameters associated with it, there is also the question of how the initial distribution of messengers and the number of messengers in the system affects the outcome. This question will not be emphasized initially, but we still present the general procedure that we used to create the initial distribution. Given m messengers, we can calculate it from a vector of independent binomial probabilities, $\hat{p} = (p_1, p_2, ...p_m)$. Here, p_i is the probability of an agent being endowed with the ith messenger. From this, the initial distribution of messengers can be computed. In general, the probability that an agent owns all the messengers

indexed by set I and does not own any messengers indexed by set J is given by

$$\prod_{i\in I} p_i \prod_{j\in J}(1-p_j)$$

More interesting, however, is the distribution of number of messengers agents use. The probability that a randomly chosen agent has k messengers is given by

$$\sum_{i_1<i_2<\cdots<i_l} p_{i_1}p_{i_2}\cdots p_{i_l}\prod_{j\neq i_k}(1-p_j)$$

With each p_i set at 0.2, the distribution of number of messengers individuals use is shown in Figure 1 (all Figures can be found in the appendix). Since the networks and the initial distribution of messengers contain random elements, the simulation was repeated 30 times for each set of parameters. The simulation was run with 10 different realizations of initial distributions on each of 3 separately generated graphs. This ensures that the results are not just statistical flukes, and allowing one to look at several runs over the same parameter region and their characteristics.

4 Results of Computational Model

4.1 Market Shares

One of the paradoxes in the messenger market is the emergence of multiple competitors in the market, in spite of strong network effects and incompatibilities. We argue that the structure of the social network can account for this feature. If users care exclusively about the choices of their neighbors in the social network, then the overall structure of the network should matter in determining how much space there is for different messengers. By means of our computational model we are able to probe this question in a controlled experiment, by changing only one parameter and looking at the effect it has on the number of messengers left in the system. In particular, we will look at the effect changing the network structure has, where we take random as the base case. Another way of decreasing connectivity in the network is by increasing the number of nodes while keeping the mean degree constant. In essence, this involves decreasing p, the probability of connection between any two nodes, by the same factor as n has been increased. By doing this, the average path length between any two nodes will be increased. What this intuitively means is that there is now more separation between people in the network, so that decisions one community of people makes have a smaller effect on other groups. Figure 2 shows the distribution of messengers surviving in the market over 30 runs of the parameter

values displayed, grouped by the type of network. We can see that there is a clear trend towards more messengers surviving when we introduce clustering via the Small Worlds graph. The effect is less pronounced when only changing the degree distribution via the scale-free graph. The cliquish structure in the small worlds graph effectively allows different messengers to be embedded in different communities in the social network. The people who connect these communities then are the ones who have more than one messenger. When increasing the number of agents in the network, while keeping the mean degree constant, we see in Figure 3 that it skews the distribution of messengers in the network to the right, making it more likely for multiple messengers to take hold in the market. This can be explained by the increase in average path length associated with the change in parameters, which makes the network effects less global in nature than in the smaller network.
It should be noted that a simultaneous increase in both C_a and C_d lead to less messengers surviving in the market. In parameter regions with low mean degree this is caused by the mere fact that many people might not have enough neighbors with the same messengers to adopt it, since people only have few neighbors. However, even with higher mean degree, the market mostly caters to a winner-take-all scenario, suggesting that the parameter values chosen could have empirical significance if the decision model people implement in reality is similar to the one presented here.

4.2 Individual Behavior

Another interesting question is under which circumstances agents use more than one messenger, and how position in the network influences the number of messengers an agent uses. It is clear without further thought that the number of messengers individuals can have is limited by the number of messengers available in the market. This said, we can look at the distribution of messengers used on an individual level, conditional on the number of messengers in the market, for the random binomial network, shown in Figure 4.
We observe that the there is an increase in the number of agents using two messengers as the amount of messengers in the entire system is increased. This is not surprising, since if there are more messengers in the market, there must also be more agents who are in between two groups that use different messengers, and hence more stable arrangements for using two messengers. On the other hand, it should be noted that when switching to a small worlds network, there is generally a lower percentage of agents than in the random binomial network who use more than two messengers. We attribute this feature to our earlier hypothesis that messengers occupy the community structure present in the small worlds network. Since there are only a few agents between these communities, only these would have to install

more than one messenger to be connected to both groups. We suspect that there are more random clusters of messengers in the random network, leading to more agents between clusters. Since the community structure of the graphs has not yet been investigated at this stage, no clear conclusion regarding the features of the data can be drawn.

We would also like to know if certain features of nodes in the network lead them to install more messengers. One obvious feature of a node that one would expect on average to contribute to the number of messengers installed is the degree, or number of neighbors, of the node. In fact, having more than twice the deletion threshold as a degree is a necessary condition for the stability of an agent having two messengers[10]. To see this, suppose that the degree of the agent, k, is $2C_a$, and she has A and B installed. Then one of the messengers would invariably get deleted when the agent is updated, since the marginal cost of at least one of the two messengers would be less than or equal to C_a.
Two other centrality measures that might be related to the number of messengers an agent uses are closeness and betweenness centrality, defined as follows:

$$\text{Closeness(node i)} = c_1 \sum_{j \neq i} d(i,j) \tag{9}$$

$$\text{Betweenness(node i)} = c_2 \sum_{i \neq j \neq k} \frac{n(j,k,i)}{n(j,k)}, \tag{10}$$

where $d(i,j)$ is the shortest distance between node i and j, $n(j,k,i)$ is the number of shortest paths from node j to k that go through i, and $n(j,k)$ is the number of shortest paths from node j to k[11]. Closeness is a measure of how near all the other agents are to a particular agent, and betweenness is a measure of what percentage of times a particular agent is on a shortest path between any two agents.
Figure 4 shows the average number of messengers an agent has installed in the last period[12] of a given simulation versus degree, betweenness, and closeness. For none of the measures is there a clear trend of how they influence the number of messengers installed. However, conditional on a high number of messengers, there is a higher probability of scoring high in the other three measures.
In Figure 5, we see the results of a linear regression analysis, where the number of

[10]Stability of an agent's state here is defined invariance under updating when all his neighbors states are held constant

[11]The terms c_1 and c_2 are normalizing constants that depend on the number of nodes in the graph

[12]The average here is taken over the 10 runs of the simulation in which the graph is held constant, and hence every agent has the same position in the network.

messengers installed is the independent variable, and various independent variables are included. We see that degree has a slight negative effect on the random graph, whereas the effect on the scale-free and small worlds graph are slightly positive. Betweenness only has a positive effect for the scale-free graph, whereas closeness has a positive effect for all graph types, although it is strongest for the random graph. The high t-values are due to the large sample size ($N = 62799$). From these results one would think that betweenness and closeness impact the number of messengers installed most, but looking back at figure 5, these measures only move in very small bounds. This shows that although they have the largest coefficient, since they only move very slightly, they will have a very low effect on the number of messengers used. On the other hand, switching the topology of the network from random binomial to the other two types will have a tangible effect, in each case raising the average number of messengers installed by about 0.6.

4.3 Efficiency

Since we observed earlier that there are less people using two messengers in the small worlds network than in the random binomial network, we might ask ourselves if the small worlds network is in some way more efficient. By the measure of what fraction of people uses more than one messenger, it certainly is, as we have found by looking at Figure 4. A more sophisticated measure of efficiency is based on the notion of overlaps of two messenger networks. For any two messenger networks N^q and N^r we can count the number of overlaps, and compare these to the number of overlaps if these networks were randomly and independently generated, given their mean degree. For any given random graph N^q, the probability of a connection between two nodes is given by

$$P(N_{ij}^q = 1) = \frac{\bar{k_q}}{n-1},$$

where $\bar{k_q}$ is the mean degree of the qth network. Given the independence assumption, we can find the probability of an overlap between two randomly generated networks q and r by

$$P(\text{Overlap between } q \text{ and } r) = P(N_{ij}^q = 1)P(N_{ij}^r = 1) = \frac{\bar{k_q}\bar{k_r}}{(n-1)^2}.$$

Since there are a total of $\frac{n(n-1)}{2}$ possible edges, the expected number and variance of overlaps is:

$$\begin{aligned} E(\text{Overlaps between } q \text{ and } r) &= \frac{n\bar{k_q}\bar{k_r}}{2(n-1)} \\ V(\text{Overlaps between } q \text{ and } r) &= \frac{n(n-1)}{2}\left(\frac{\bar{k_q}\bar{k_r}}{(n-1)^2}\right)\left(1-\frac{\bar{k_q}\bar{k_r}}{(n-1)^2}\right) \end{aligned}$$

Our efficiency measure now counts the actual number of overlaps between any pair of messenger networks in each simulation run, subtracts from it the expected number of overlaps, and then divides it by the standard deviation. The negative sign in the formula is meant to make a higher score mean less overlaps than expected, which in efficiency terms would be a good outcome. The scores are then averaged over all possible permutations, and over multiple simulation runs. The efficiency is denoted by e, and the number of overlaps between network i and j by o_{ij}.

$$e = \binom{m}{2}^{-1} \sum_{i<j} -\frac{o_{ij} - E(o_{ij})}{\sqrt{V(o_{ij})}}$$

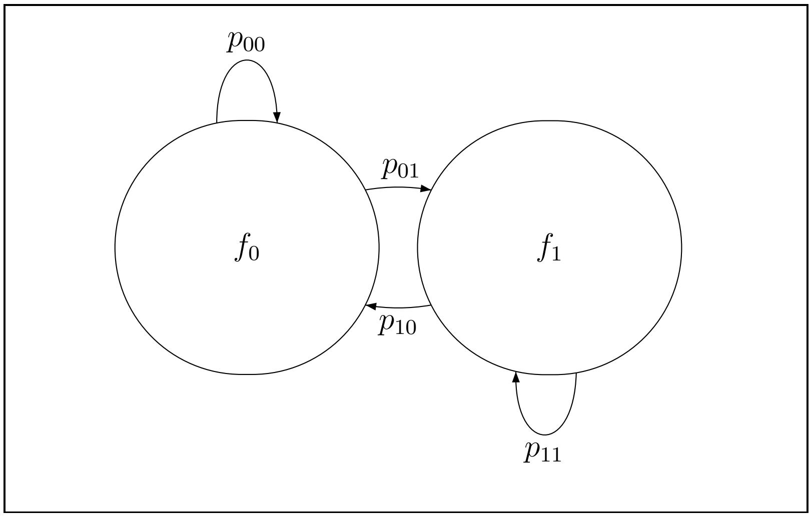
In Figure 7, efficiency is plotted against mean degree. We observe that there is a clear trend for more variability in efficiency as mean degree is increased, and also for efficiency to generally decrease. There is no clear difference visible in efficiency as a function of the network structure, at least as far as our measure can tell.

5 Mathematical Model

After noticing some of the regularities in the computational model, a simple mathematical model was devised in order to predict certain outcomes and to allow more precise analysis of the diffusion process. One question of interest is whether a model lacking network structure can replicate results in the random network, since this would suggest that this implicit assumption on the network structure has been made when studying network goods without including the network structure.

5.1 One Messenger Model

To abstract away the network structure, we make the following two assumptions. First, we assume that every agent in the network has average degree, $\bar{k}$. Furthermore, we assume homogeneity of neighbors in the network, meaning that the fraction of users having one messenger in the entire network equals the fraction of users having one messenger in the neighborhood of any agent. With these two simplifying assumptions, we start with a model that includes two states, either having a messenger, or not having a messenger. The following state transition diagram then applies:



With the assumptions in mind, we can now formulate a simple model of adoption dynamics. The probability of an agent in state f_0 adopting a messenger is given by the probability that at least C_a of its neighbors have a messenger, i.e. are in state f_1. Similarly, the probability that an agent in state f_1 deletes a messenger is given by the probability that less than or equal to C_d of its neighbors are in state f_1. This produces the following set of transition probabilities

$$P(f_0 \rightarrow f_1) = \ p_{01} = \sum_{k=C_a}^{\bar{k}} \binom{\bar{k}}{k} f_1^k (1-f_1)^{\bar{k}-k} \tag{11}$$

$$P(f_1 \rightarrow f_0) = \ p_{10} = \sum_{k=0}^{C_d} \binom{\bar{k}}{k} f_1^k (1-f_1)^{\bar{k}-k} \tag{12}$$

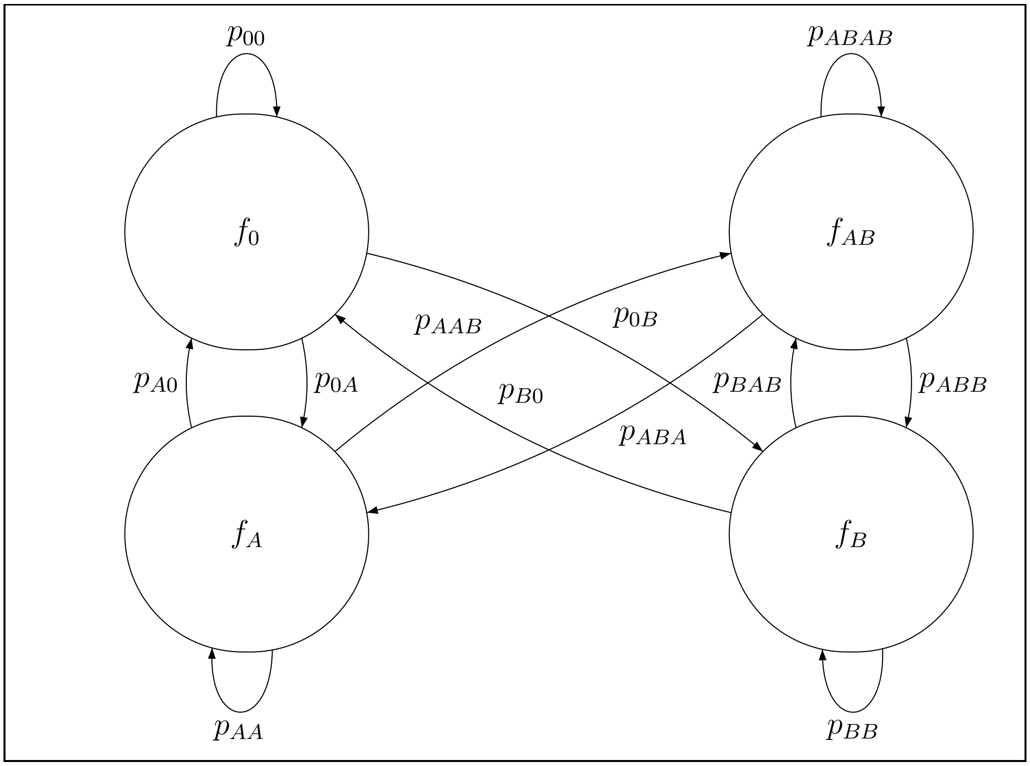
From this, we can generate a set of difference equations that generates the expected fraction of agents in each state for period t, f_i^t, given f_i^{t-1}, the fractions at the previous time step.

$$\begin{aligned} E(f_1^t) &= f_1^{t-1}(1-p_{10}) + f_0^{t-1} p_{01} \\ &= f_1^{t-1}(1-p_{10}) + (1-f_1^{t-1}) p_{01} \\ &= f_1^{t-1}(1-p_{10}-p_{01}) + p_{01} \qquad (13) \\ E(f_0^t) &= 1 - E(f_1^t) \qquad (14) \end{aligned}$$

With initial conditions f_0^0 andf_1^0 and parameters C_a, C_d, and $\bar{k}$, the expectation of f_0 and f_1 in period t can be found by iteration. Two sample trajectories, which illustrate the typical qualitative behavior of the system, are shown in figure 8.
We observe that after a few periods all agents either have zero messengers or one messenger. This is mostly due to the strong positive feedback in the system, since if more people adopt, the probability of adoption increases, since the transition probabilities depend positively on f_1. The same holds for the case where agents delete messengers, since the probability of adoption is negatively related to f_0. Comparing the mathematical model with the computational one when run with one messenger only it becomes apparent that the dynamics are not quantitatively predictive. However, when looking at one qualitative feature of the model, namely the parameter values that separate the behavioral regimes of most people using zero messengers and most people using one messenger, the mathematical model makes precise predictions.

5.2 Two Messenger Model

With the one messenger model it is only possible to predict a very simple feature of the model. We would, however, also like to be able to mathematically model and analyze the initial leap in people using two messengers, which is a pervasive feature of many simulation runs. Also, it would be of interest to see whether independent of the network structure we could still see the emergence of two messengers in the market. In order to approach these two questions, we made the same simplifying assumptions as in the one messenger model. An agent can now be in one of four states, either having no messenger, messenger A, messenger B, or both messenger A and B, which is denoted by AB. The fraction of people in state A and state B can then be added up to obtain the fraction of people using one messenger. The fraction of people using two messengers is then simply given by f_{AB}. These dynamics give rise to the following state transition diagram:



The transition probabilities for this model tend to be more involved, which is why only one is explicitly derived here, while the other ones can be looked up in the Appendix. Since there are more states neighboring agents can be in, the probabilities are now given by a multinomial distribution instead of a binomial distribution. The probability of a neighbors being in state A, b neighbors being in state B and c neighbors being state AB[13] is given by

$$P(A = a, B = b, AB = c) = \frac{k!}{a!b!c!(\bar{k} - a - b - c)} f_A^a f_B^b f_{AB}^c (1 - f_A - f_B - f_{AB})^{\bar{k}-a-b-c}$$

Furthermore, since there are many combinations in which events can happen, the sums tend to have more complicated indices than in the one messenger case. For instance, if an agent has zero messengers, he would adopt A if at least C_a of her neighbors have A (or AB) and less neighbors have B (or AB) than A. So, for instance, if C_a is 4, and an agent has 3 neighbors with A, five neighbors with AB, and 1 neighbor with B, then effectively this would be counted as being able to talk to 6 neighbors when adopting B and 8 neighbors when adopting A, so A would be adopted since it exceeds C_a and maximizes the number of people the agent could talk to. There is also the case of ties, in which the number of neighbors that have A

[13] Implicit in this probability of course is that $\bar{k} - a - b - c$ neighbors are in state 0.

is equal to the number of neighbors that have B. This can only happen if the number of neighbors having any one of them is no greater than $\lfloor\frac{\bar{k}}{2}\rfloor$[14]. Given that there are a neighbors in state A and B, respectively, there can only be up to $\bar{k}-2a$ neighbors in state AB, since there are only a total of $\bar{k}$ neighbors by our first assumption. An additional constraint is given by the requirement that we need there to be at least C_a neighbors to consider adoption, hence neighbors in state AB should contribute at least the remaining neighbors necessary to add up to C_a, hence the neighbors in state AB in a tie should at least be $\max(0, C_a - a)$. From these constraints, we can generate the probability of a tie between switching to state A and state B, given that an agent is in state 0[15]:

$$P(\text{Ties}(A,B)|0) = \sum_{a=0}^{\lfloor\frac{\bar{k}}{2}\rfloor} \sum_{c=\max(0,C_a-a)}^{\bar{k}-2a} P(A=a, B=a, AB=c)$$

Keeping in mind earlier comments about which conditions have to hold true when switching to state A from state 0, we can no write down the probability of this event happening as:

$$P(0 \to A) = \sum_{a=1}^{\bar{k}} \sum_{b=0}^{\min(a-1,\bar{k}-a)} \sum_{c=\max(C_a-a,0)}^{\max(\bar{k}-a-b,0)} P(A=a, B=b, AB=c) + 0.5P(\text{Ties}(A,B)|0)$$

The $P(\text{Ties})$ term needs to be included since in the case of a tie between A and B, there is a 50 per cent chance of A being selected.
With the transition probabilities defined, we can now define the dynamics of the system by the following matrix equation:

$$E(f^t) = \mathbf{T} f^{t-1},$$

where $f^t = (f_0^t, f_A^t, f_B^t, f_{AB}^t)$ and $\mathbf{T}$ is the transition matrix, which depends on f^{t-1} via the transition probabilities[16]. To test in how far this more advanced model captures the dynamics of the computational model, we repeatedly iterated this model

[14]Here $\lfloor x \rfloor$ denotes the floor function of x, i.e. the greatest integer n such that n is less than or equal to x.

[15]In a slight abuse of notation, we use $P(\text{Ties}(A,B)|0)$ not to denote a conditional probability, but rather to distinguish this tie from the one noted in the appendix between going from state AB to state A and state B.

[16]$\mathbf{T}$ can be found explicitly written out in the appendix

with the same initial conditions as the computational model (with 2 messengers instead of 5), and looked at the resulting trajectories of f_0, $f_1$andf_2. The mathematical model turned out to be very close in dynamics to the computational model, especially in the case of the random network. When other network structures were used, the resemblance was not as striking between trajectories, which suggests further that the network structure should be included in a mathematical model. Another point that should be made is that the only time the mathematical system reached a fixed point with 2 messengers, was when both messengers started out with exactly the same initial fraction of users, i.e $f_A = f_B$. This result is not very robust, since the equilibrium is unstable, so there is no indication on how to make more than two messengers emerge with this model without the network structure included.

6 Further Directions

We would like to further investigate the community structure in the social networks induced by the messenger networks, and its correlation with the underlying community structure. We also seek to make connections with the existing network formation literature in economics (see for example [7]), and see if there are existing frameworks that can be applied in this project. We would furthermore like to do some more detailed analytical work with the decision model that we developed, and extend the model to deleting and adopting more than one messenger in the same period. To test for robustness of our results, we would like to experiment with other, perhaps more realistic, decision algorithms and see if it still yields the same conclusions. It would also be worth investigating how one could strategically tip the market towards an outcome where one messenger dominates, by making certain central agents in the network switch their messenger. Finally, it would be interesting to see if there is a position measure that predicts the number of messengers more clearly than the ones we have investigated.

7 Conclusions

We have shown that within a simple model of technology adoption, the explicit introduction of social network structure has lead to the emergence of more than one technology taking a hold in the market. Furthermore, there is empirical evidence that this scenario is occurring in the instant messenger market, despite expert predictions that it would turn out to be a winner-take-all market. We have demonstrated that this only occurs in markets where the connectivity between people is sufficiently high and random. We hope to have solved the paradox in the initial messenger market, and emphasize the importance of social connections in economic environments. Potential market entrants should consider the structure of the network relevant to their product when making decisions about product placement. On the other hand, policymakers should be aware of the structure of the network and the possible outcomes when deciding on whether to intervene in a technology market where network externalities play a role. Not always will incompatibility of technologies lead to a monopolistic outcome, as instant messengers have taught us.

Acknowledgements

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Appendix

7.1 Transition Probabilities for the two Messenger Model

We now list all the transition probabilities used in the two messenger mathematical model, except for the ones already listed before[17]:

$$
\begin{aligned}
P(0 \to B) &= \sum_{b=1}^{\bar{k}} \sum_{a=0}^{\min(b-1,\bar{k}-b)} \sum_{c=\max(C_a-b,0)}^{\max(\bar{k}-a-b,0)} P(A=a, B=b, AB=c) \\
&+ 0.5P(\text{Ties}(A,B)|0) \\
P(A \to A) &= \sum_{a=0}^{\bar{k}} \sum_{c=\max(C_d+1-a,0)}^{\bar{k}-a} \sum_{b=0)}^{\min(C_a-1,\bar{k}-a-c)} P(A=a, B=b, AB=c) \\
P(A \to 0) &= \sum_{a=0}^{C_d} \sum_{c=0}^{C_d-a} \sum_{b=0}^{\min(C_a-1,\bar{k}-a-c)} P(A=a, B=b, AB=c) \\
&+ 0.5P(\text{Ties(AB,0)}) \\
P(A \to AB) &= \sum_{a=0}^{\bar{k}-C_a} \sum_{c=\max(C_d+1-a,0)}^{\max(\bar{k}-a-C_a,0)} \sum_{b=C_a}^{\bar{k}-a-c} P(A=a, B=b, AB=c) \\
&+ 0.5P(\text{Ties(AB,0)}) \\
P(\text{Ties(AB,0)}) &= \sum_{a=0}^{\min(C_d,\bar{k}-C_a)} \sum_{c=0}^{\min(C_d-a,\max(\bar{k}-C_a-a,0))} \sum_{b=C_a}^{\bar{k}-a-c} P(A=a, B=b, AB=c) \\
P(AB \to AB) &= \sum_{a=C_d+1}^{\bar{k}-C_d-1} \sum_{b=C_d+1}^{\bar{k}-a} \sum_{c=0}^{\bar{k}-a-b} P(A=a, B=b, AB=c) \\
P(AB \to B) &= \sum_{b=C_d+1}^{\bar{k}} \sum_{a=0}^{\min(\bar{k}-b,C_d)} \sum_{c=0}^{\bar{k}-a-b} P(A=a, B=b, AB=c) \\
&+ 0.5P(\text{Ties}(A,B)|AB) \\
P(AB \to A) &= \sum_{a=C_d+1}^{\bar{k}} \sum_{b=0}^{\min(\bar{k}-a,C_d)} \sum_{c=0}^{\bar{k}-a-b} P(A=a, B=b, AB=c) \\
&+ 0.5P(\text{Ties}(A,B)|AB) \\
P(\text{Ties}(A,B)|AB) &= \sum_{a=0}^{C_d} \sum_{b=0}^{\min(\bar{k}-a,C_d)} \sum_{c=0}^{\bar{k}-a-b} P(A=a, B=b, AB=c)
\end{aligned}
$$

[17] The transition probabilities from state B are defined analogous to the ones listed for state A.

The transition matrix for the two messenger model is given by

$$\mathbf{T} = \begin{pmatrix} 1-0.5(p_{0A}+p_{0B}) & 0.5p_{A0} & 0.5p_{b0} & 0 \\ 0.5p_{0A} & 1-0.5(p_{A0}+p_{AAB}) & 0 & 0.5p_{ABA} \\ 0.5p_{0B} & 0 & 1-0.5(p_{B0}+p_{BAB}) & 0.5p_{ABB} \\ 0 & 0.5p_{AAB} & 0.5p_{BAB} & 1-0.5(p_{AAB}+p_{BAB}) \end{pmatrix}$$

7.2 Figures

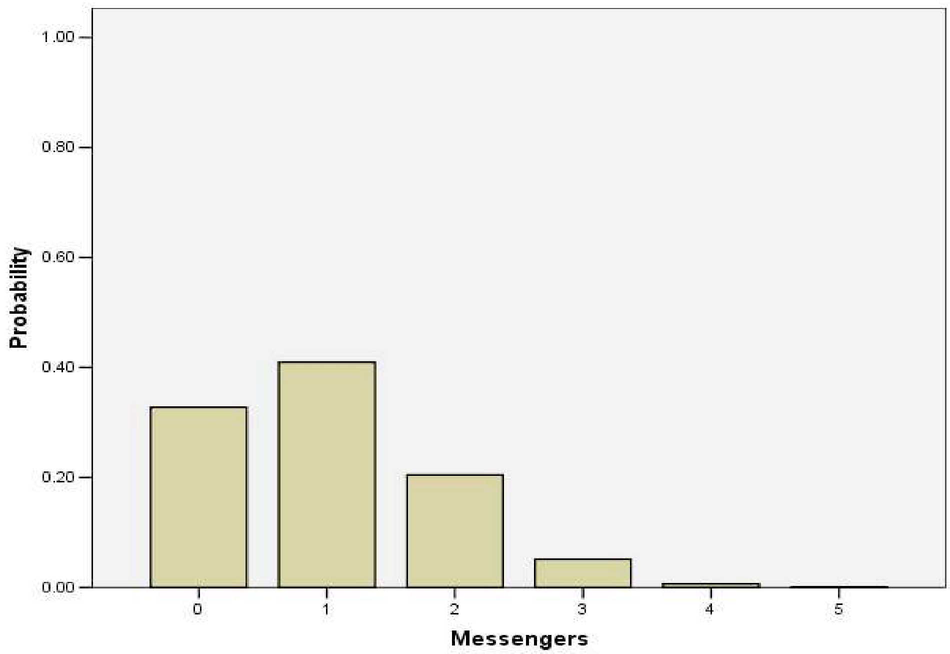
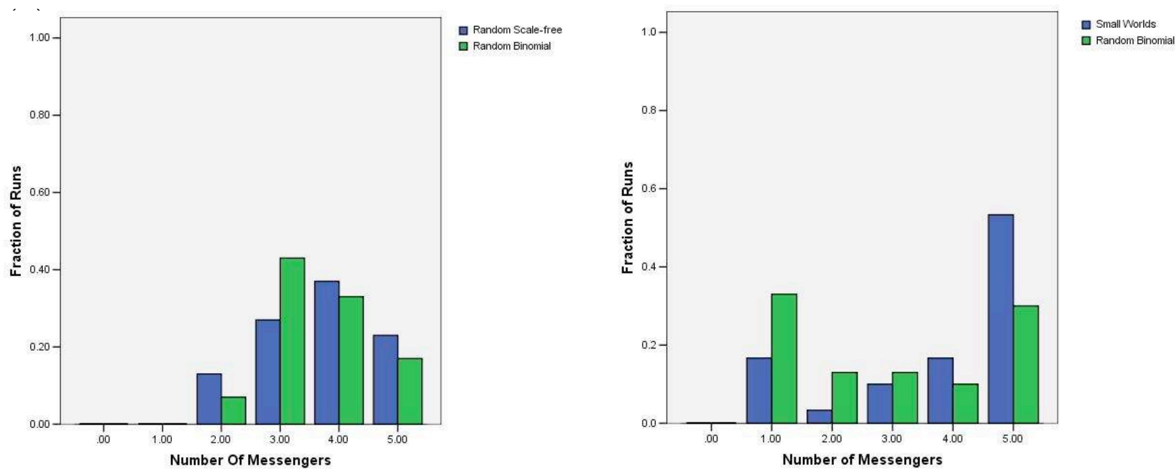


Figure 1: Initial Distribution of Number of Messengers Used by Individuals



$\bar{k} = 5$ $\bar{k} = 10$

Figure 2: $C_a = 3, C_d = 1, n = 200$

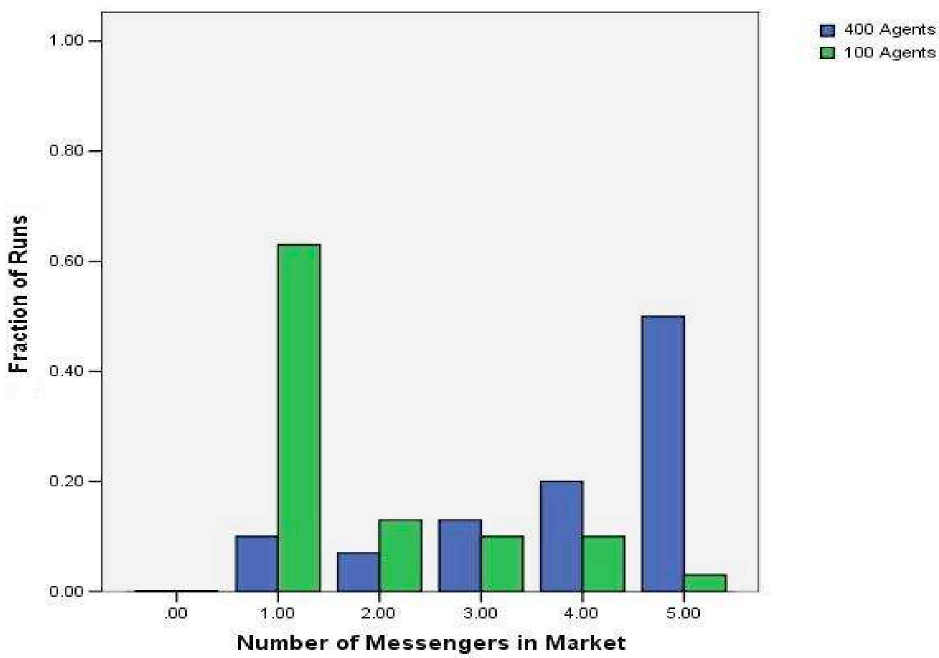
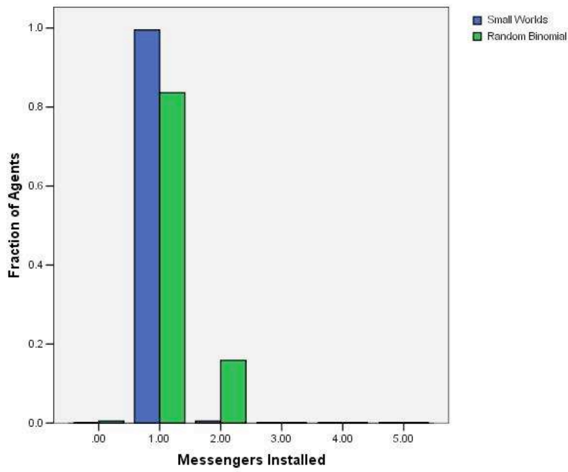
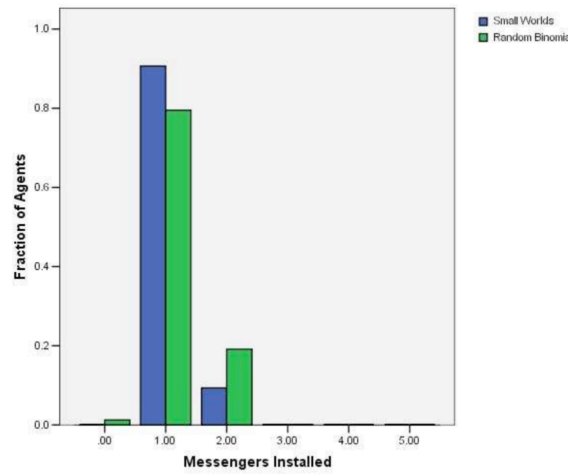


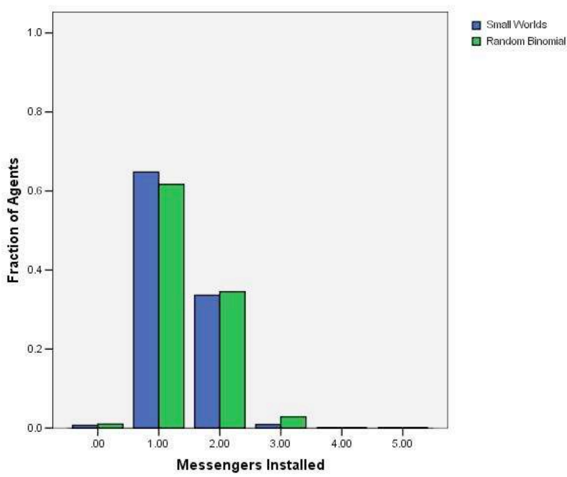
Figure 3: $C_a = 4, C_d = 1, \bar{k} = 10$, Network = Random Binomial



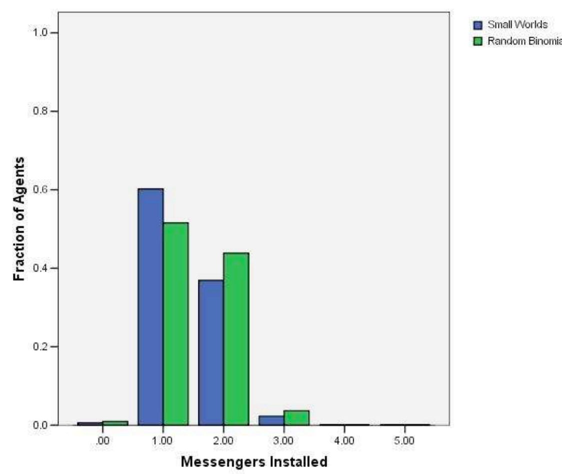
2 Messengers in Market



3 Messengers in Market



4 Messengers in Market



5 Messengers in Market

Figure 4: Number of Messengers installed by Network and Messengers in Market, $C_a = 3, C_d = 1, \bar{k} = 10, n = 200$

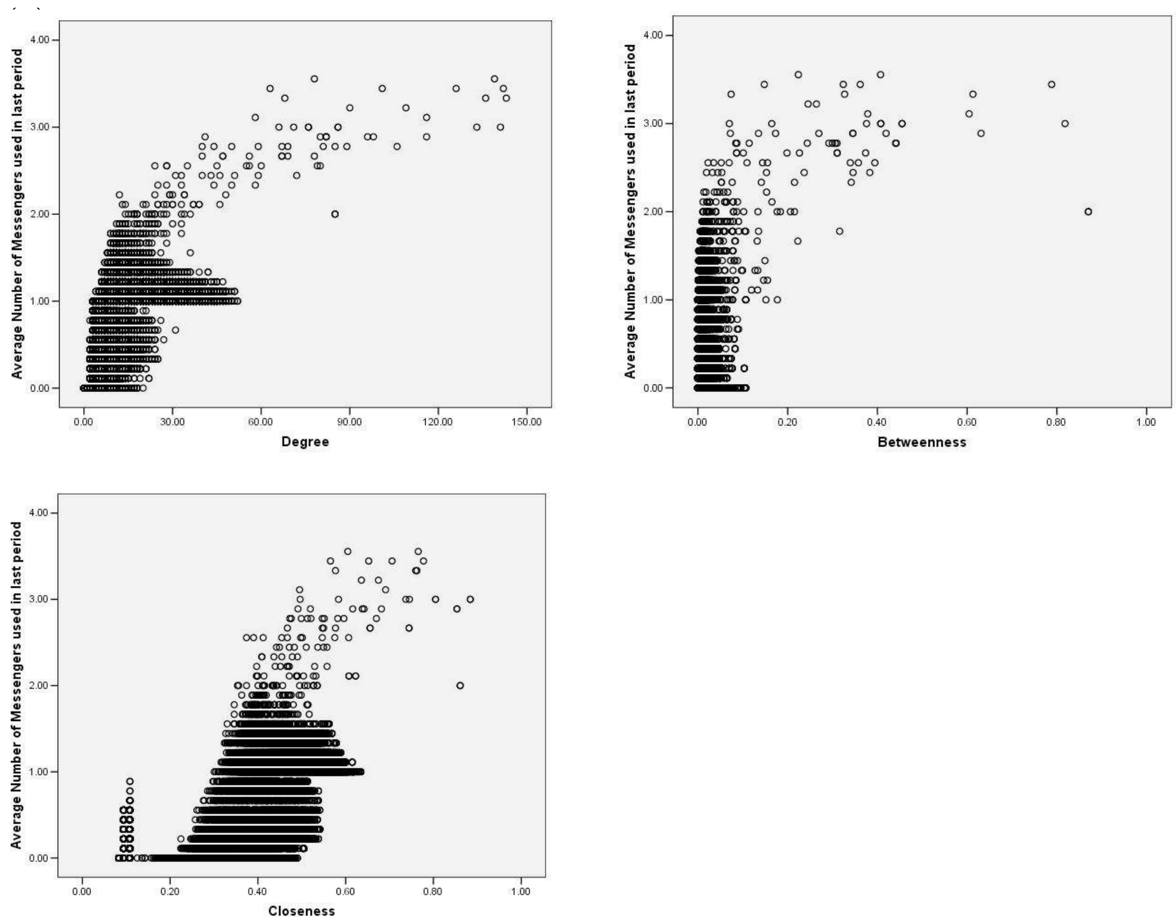


Figure 5: Node Characteristics vs Messengers Used

Coefficients[a]

Model		Unstandardized Coefficients B	Std. Error	t	p
1	(Constant)	-.592	.016	-36.740	.000
	Degree	-.013	.000	-32.120	.000
	Adoption Threshold	-.121	.001	-118.702	.000
	Number of Agents	.001	.000	45.346	.000
	Betweenness	-3.364	.246	-13.657	.000
	Closeness	4.213	.035	121.005	.000
	MeanDegree	.004	.000	11.455	.000
	Scale-free Graph	.688	.015	46.776	.000
	Small Worlds Graph	.613	.015	40.197	.000
	Small Worlds * Betweenness	2.836	.257	11.051	.000
	Scale-free * Betweenness	4.387	.296	14.819	.000
	Small Worlds * Closeness	-1.912	.043	-44.429	.000
	Scale-free * Closeness	-2.719	.044	-61.323	.000
	Small Worlds * Degree	.016	.000	37.705	.000
	Scale-free * Degree	.040	.001	53.042	.000

a. Dependent Variable: Messengers Installed

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.822	.675	.675	.25778

Figure 6: Regression Results

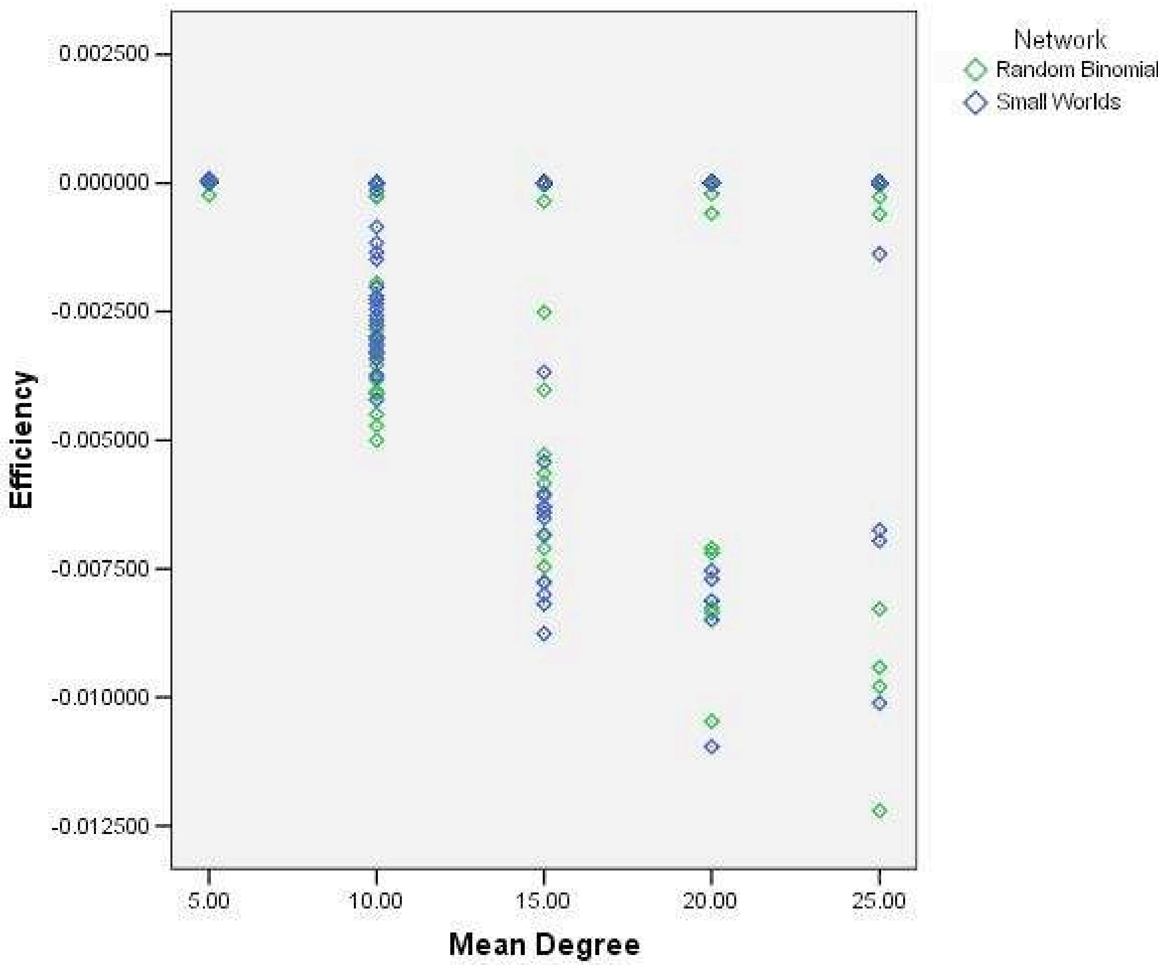


Figure 7: Efficiency, $C_a = 3, C_d = 1, n = 200$

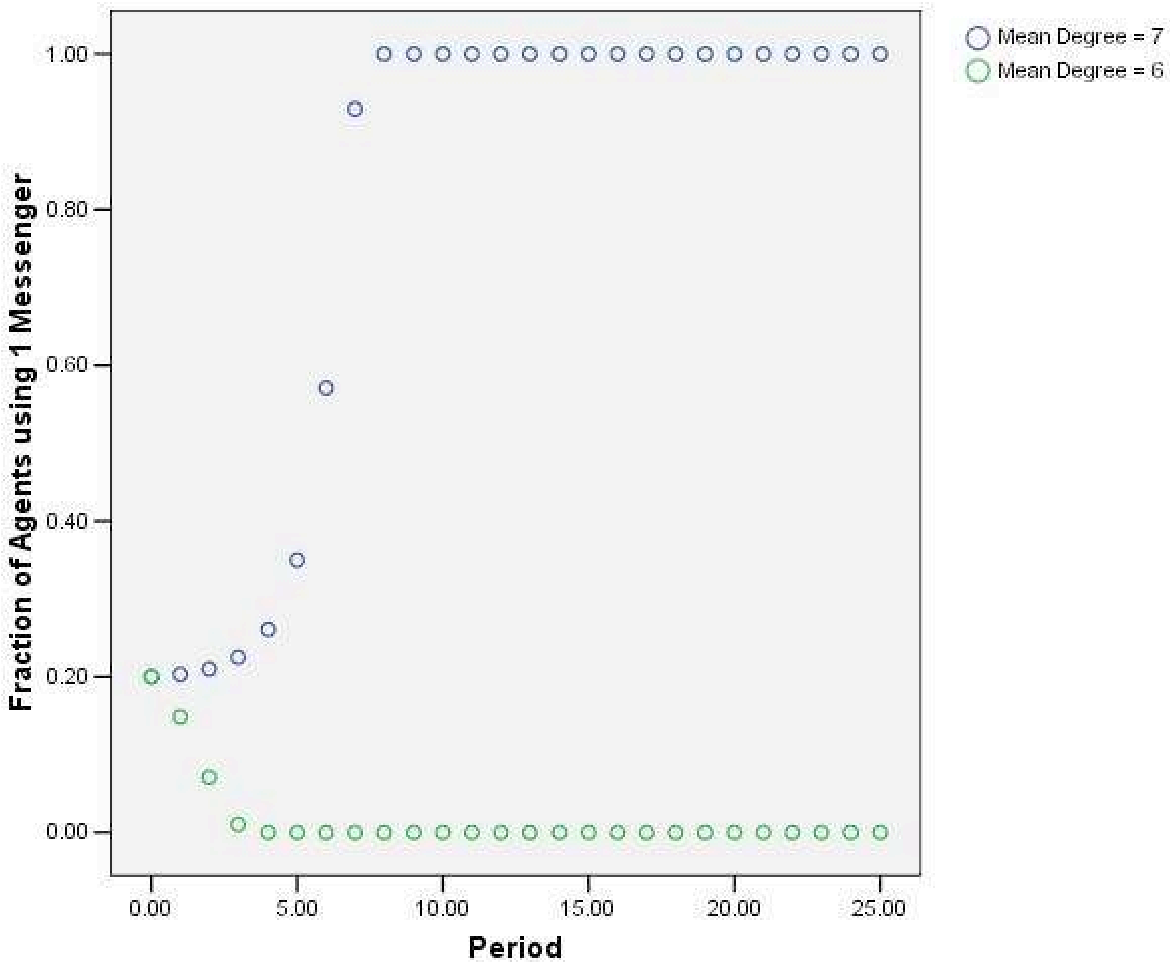


Figure 8: Sample Trajectories for One Messenger Model, $C_a = 3, C_d = 1, f_1^0 = 0.2$