# Simulating the Driving Forces of Long-run Inequality

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Abstract

A simulation of a model based on the theoretical work developed in Bowles (2005) is presented. It is shown how inheritance of wealth, positive assortation, constraints to investment, taxes, genetically-inherited income factors and other cultural factors transmitted by parents and society combine to generate stationary inequality. A study of comparative statics with a modern capitalist society as the benchmark case is discussed. It is argued that more egalitarian societies are possible if the most important source of wealth is human capital and human capital formation is left to society and not to parents, i.e. states that provide high quality and widespread institutions.

## 1. Introduction

Humanity has made great advances in productivity and technology; we are now capable of generating more wealth than ever before. Nevertheless, we have not achieved much in terms of equality we live in unequal societies in an unequal world. What are the forces driving inequality and how do they combine to reach a stationary level? These are the questions addressed in the present work.

It is first argued that inheritance plays a crucial role in inequality. People not only inherit material capital but also human capital, which is genetically and culturally transmitted by parents and to some extent by society. Parents pass on their genes and with them some income-enhancing factors, some independent of skill such as race, looks and height and others related to skill such as IQ and other cognitive and physical abilities. Wolff (2004) reports for the U.S. that in 2001 the mean net worth of Hispanics and African-Americans was 17% and 14% respectively of the net worth of Whites. There is also evidence that other non-skill factors (looks, height and obesity) are predictors of

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earnings, see Hammermesh and Biddle (1993) and Bowles, Gintis and Osbourne (2002). Bowles and Gintis (2002) also argue that the genetic transmission of IQ seems to be relatively unimportant in the intergenerational transmission of economic status.

Parents also transfer family culture to their children, e.g. eating and safety habits, good manners and moral values. Bowles and Gintis (2002) survey some empirical evidence where social traits as fatalism, deviant behavior and occupational self-direction are transferred by parents to their children and in some cases, e.g. fatalism (Osborne, 2005), this has an effect on children's earnings. There are also some factors that are a combination of genetic and cultural transmission such as emotional intelligence and other behavioral skills.

People also face constraints to investment. Minimum required investment is ubiquitous due to project size and policies within investment firms. There are also constraints to credit access that prevent the poor and illiquid from investing. The poor may not have an income high enough to save an accumulate the required amounts for investment Wolff (2004) shows that in 2001 96.1% of U.S. households with an income higher than 250,000 held stocks while only 12.5% with an income under 15,000 did. This provides some evidence that there are constraints to investment and that they are separating the pure consumers from investors.

Another source of inequality frequently seen in societies is positive assortation. People are more likely to enter long term relationships such as marriage, friendship or investment relations with people they regard as similar and compatible in terms of human and material capital, e.g. education, status, wealth and race among many others. The relation between wealth and the number of children is also studied. It is believed that when wealth and offspring are positively related and thus the rich have more children than the poor wealth is better distributed through more equal material bequests.

Finally, taxes are considered as the main explicit redistributive force and modeled with a progressive tax function. Volatility, randomness and luck are introduced in the model via shocks to the generation of wealth and human capital. It is important to include randomness because it alone can generate inequality. Carter (2005) simulates a simple exchange economy where agents start with the same initial wealth and randomly choose an agent to give one dollar to. Contrary to expected, the distribution of wealth is a Boltzmann distribution instead of a Normal distribution, indicating that inequality can arise simply by randomness and luck; some agents get lucky and do better than others. Mookherjee and Ray (2003) distinguish models in the inequality literature where markets are inherently equalizing and inequality is uniquely explained by shocks and randomness, namely, Becker and Tomes (1979) and Loury (1981).

The literature addressing inequality is abundant. Works similar in spirit to what is presented here are as follows. Nirei and Souma (2004) construct a two factor stochastic model of labor and asset income that can explain the income distribution drawn from the individual tax returns data in the U.S. and Japan for 40 years. The income distribution is exponential for the low and middle part while the tail exhibits a power law. They are able to reproduce the exponential decay of the middle region with an additive process for labor income and reproduce the fat tail with a multiplicative process for asset income. Despite the fact that the two-factor stochastic model fits the empirical data well, it does not account explicitly for the different forces driving inequality.

Mookherjee and Ray (2002) start a literature where persistent inequality can be explained by fundamental non-stochastic reasons. They present a model where dynasties decide how much to consume, how much to spend in the next generation's occupational training (e.g. education) and the amount of a financial bequest in an economy with imperfect credit markets. They show that because different professional categories must be occupied for the economy to function, even identical agents will behave non-symmetrically and end up in different professions with different incomes. If the bequest motive is not so strong and thus parents of low-income profession agents do not compensate them with high enough financial bequests economic inequality will arise and persist.

Mookherjje and Ray (2003) present a work in the same vein where the relative returns to different professions depend on the occupational decisions made by other agents generating what they denominate *pecuniary externalities* in investment. They show that inequality is inevitable, when credit markets are imperfect and there is some occupational diversity, not only in incomes but in consumption and utility. They also show that inequality is path dependant when profession possibilities are not enough as to be completely divisible, i.e. divisibility in the set of investment opportunities.

Mookherjee and Ray (2004) construct a model that generalizes the three types of models that they identify in the literature, namely, *exogenous inequality* where markets are equalizing and any inequality is explained via shocks and randomness; *endogenous inequality* where markets are inherently un-equalizing (as in the models previously discussed) and *neutral towards inequality* models where there are both equal and unequal steady states whose occurrence depend on the initial conditions and history of the society. The model maintains the imperfect credit market and the non-perfect substitutability of profession inputs of the previous models. They show that the condition to establish whether a market is equalizing or un-equalizing is the relation between the number of occupational possibilities and the strength of the bequest motive. If occupational differences are suppressed by financial bequests then the market is equalizing, conversely, if the range of training costs is wide enough compared to the bequest incentive then the market is un-equalizing.

All the previous models stress the importance of the occupational structure and imperfection of capital markets but do not account for other social and economic forces such as taxes, positive assortation, reproduction and different possibilities within the inheritance of human capital. The following work attempts to shed some light into those aspects.

The present model consists of a simulation with agents that interact and behave according to some predetermined rules. Albeit the use of agents in the simulation, it can not be construed as an agent-based model since agents can neither learn nor change their behavior according to their personal experience and interaction with other agents. Agents are used for the ease of the simulation and as a better representation of reality. All the results depend entirely on the particular equations and methods used and hence are determined in advance, i.e. agent behavior leading to global results regarding inequality is totally predetermined and does not evolve in time. Despite the deterministic character of the simulation it is the only way to gain insight into the behavior of the model since a complete analytical solution without any simplifications is unattainable. The simulation was programmed in Java using the RePast[2] libraries.

In section 2 the structure of the model and simulation will be explained. Section 3 discusses the benchmark case of a modern capitalist society and presents the results of the comparative statics, while section 4 will conclude.

## 2. The Model

There is a fixed amount of agents divided equally into two groups (this can be thought of as men and women but gender is not an issue in the model). Each agent is defined basically by three variables: the agent's material bequest, its level of human capital, and an assortation measure that is used to rank agents in both groups for future assortative matching. Agents start with equal levels of material bequest and human capital. The first step in the simulation is to find a partner according to the assortation process, agents then turn into couples whose material bequest and human capital are the sum of the material bequest and human capital of each of the partners. The couple then proceeds to consume part of its material bequest and invests the remaining amount. The couple's final wealth is a function of the material bequest remaining after consumption (amount to invest) and the couple's human capital. The couple then reproduces and passes on its wealth to its offspring. Children receive their parents' wealth as their material bequest. Then, the new generation starts another iteration by finding a partner. This process continues until the end of the simulation.

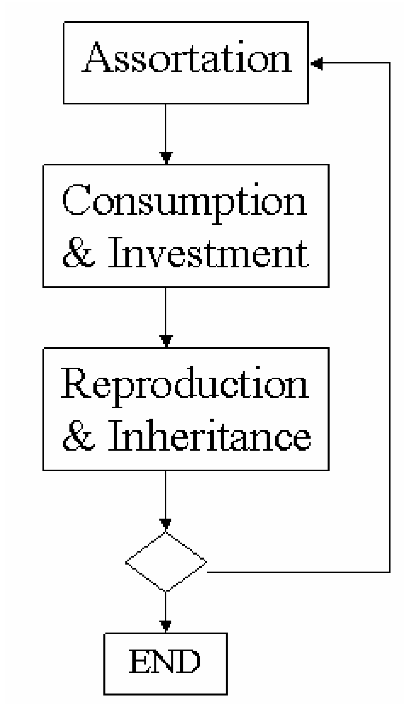



**Figure 1: Simulation Structure**

[2] Recursive Porous Agent Simulation. http://repast.sourceforge.net/

## Inheritance of Material Capital (B)

The wealth of the parents is given as a material capital bequest to the children. The inheritance of wealth can happen under two scenarios: A society where all the family's wealth is left to the first child (primogeniture) and nothing is left to the remaining children and a society where every son gets the same (equal inheritance). In both cases the wealth of the parents is taxed before it is passed on to their descendants.

$$
\begin{aligned}
t &: \text{Tax} \\
W_i^c &: \text{Wealth of i-th couple} \\
B_{ji} &: \text{Bequest of j-th son of the i-th couple} \\
n_i &: \text{Number of kids of the i-th couple}
\end{aligned}
$$

**Primogeniture:**

$$
\begin{aligned}
B_{1i} &= W_i^c(1-t) \\
B_{ji} &= 0 \quad \forall\, j \neq 1
\end{aligned}
$$

**Equal Inheritance:**

$$
B_{ji} = \frac{W_i^c(1-t)}{n_i} \quad \forall\, j
$$

## Inheritance of Human Capital (H)

Human capital is a variable that gathers all the income-generating factors different from material capital. The inheritance of human capital is divided in two parts; one part is directly inherited by parents and the other is given by society. The parameter λ permits changing the relative importance of society and parents in the formation of human capital in the next generation, higher λs give more importance to parents in the creation of human capital. It is important to differentiate between human capital given by society and that inherited from the parents in order to analyze the effect of societies that provide the necessary institutions to build human capital independent of the child's initial conditions, e.g. public education, health care, meritocracy and enforcement of equal-opportunity employment among others.

The part of human capital inherited from parents consists of two parts; the first one captures the inheritance of income-generating factors that are genetically transmitted such as race, looks and height and family-culturally transmitted such as good manners, eating, hygienic and safety habits, and a combination of genes and family culture such as emotional intelligence and other social skills. This is modeled by children receiving a combination of their parents' human capital according to a random proportion *b*. The second part relates to those income-generating factors that depend on the parents' wealth.

Wealthier parents are more likely to have better connections, memberships (clubs, extracurricular education), pay better education and health care. The parameter *z* allows changing the relative importance in the generation of income of the genetically inherited factors and those depending on wealth. A low value of *z* means that no matter how wealthy parents are they can only improve their children's income-generating capabilities in the genes they pass on. Note that if $\lambda$ is low not even then race or height will give advantage to some kids since a low $\lambda$ means that society provides institutions that suppress such differences.

Society contributes to the formation of human capital through two different mechanisms. The first is represented as the average level of human capital and is a reversion to the mean. It can be understood as all the intangible institutions that suppress differences in human capital: mainly laws and public initiatives. The second part consists of the taxes taken from the couples' wealth right before inheritance. It can be understood as all the tangible institutions that suppress differences in human capital, e.g. public schools and hospitals. Note that the next generation receives the taxes not as a part of their material bequest but as part of the contribution of society to their human capital.

Finally, there is a shock simulating volatility and whatever randomness there is involved in human capital formation. Every individual is different and even if human capital came entirely from society every person would still have different levels of human capital.

| | | |
|---|---|---|
| $\overline{W}$ | : | Mean wealth |
| $\overline{H}$ | : | Mean H |
| $H_k^i$ | : | H of i-parent k-th generation |
| $H_k^j$ | : | H of j-parent k-th generation |
| $\varepsilon$ | : | shock |
| $b$ | $\sim$ | random[0,1] |

$$H_{k+1} = \lambda[(1-z)(bH_k^i + (1-b)H_k^j) + zW_k^c] + (1-\lambda)[\overline{H} + t\overline{W}] + \varepsilon_1$$

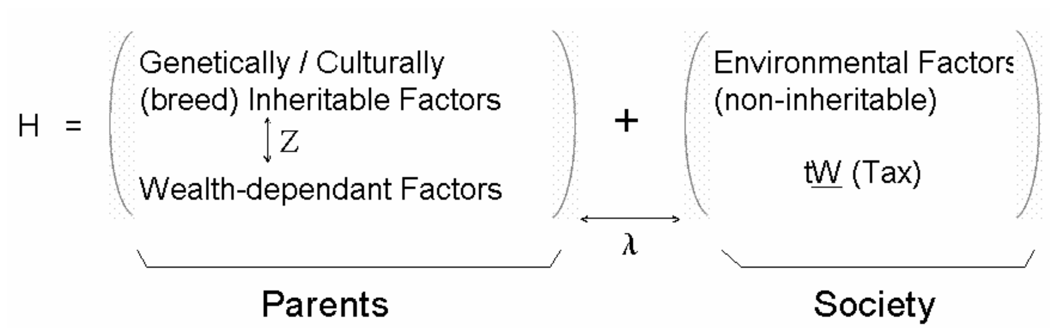



**Figure 2: Human Capital Composition**

## Assortation

Each agent has an assortation measure that is a function of its material and human capital *B* and *H* respectively. At the beginning of each iteration agents in both groups (male and female) are sorted according to their assortation measure. Every agent from one group then chooses with probability *r* an agent on the other group that has the same rank as his own and with probability 1-*r* it chooses a partner at random.

| | | |
|---|---|---|
| $r$ | : | Assortation probability |
| $A_i$ | : | i-th agent's assortation measurement |

$$r \Rightarrow A_i = B_i^{\gamma} H_i^{1-\gamma}$$
$$1 - r \Rightarrow \text{Random}$$

## Consumption and Investment

Once couples are formed they proceed to consume part of their material capital *B*. Consumption is a constant fraction of the couple's material capital. The parameter *c* determines how much material capital is dedicated to consumption and therefore how much is left for investment. A value of 0.3 for *c* would mean that 30% of the material capital is consumed. Additionally there is a normal consumption, which is the minimum that every couple has to consume before dedicating material capital to investment. The normal consumption is defined by the exogenous parameter *Kc* multiplied by the mean wealth $\underline{W}$, which is an endogenous variable; if *Kc* is equal to one then the minimum consumption is exactly equal to the average wealth. Every couple whose fraction of consumption dictated by *c* is not enough to consume the minimum required will have to consume a higher percentage of its material capital until the normal consumption is reached. Note that if a couple's material capital is lower or equal than the normal consumption the material capital is totally consumed leaving nothing for investment. Normal consumption ($Kc\underline{W}$) can be understood as a threshold that separates pure consumers from investors. This serves as a way to simulate the constraints to investment previously discussed.

| | | |
|---|---|---|
| $B_c$ | : | Couple Bequest |
| $B_c$ | = | $B^i + B^j$ |
| $B'_c$ | : | Couple bequest after consumption |
| $I$ | : | Investment |
| $K_c$ | : | Normal consumption coefficient |
| $c$ | : | $\epsilon[0,1]$Fraction of Bequest consumed in excess of $K_c$ |

$$
\begin{aligned}
\textbf{if}(B_c <= K_c\overline{W}) \quad B'_c &= 0 \quad \text{consume entire bequest} \\
\textbf{else if}(cB_c < K_c\overline{W}) \quad B'_c &= B_c - K_c\overline{W} \\
\textbf{else}(cB_c >= K_c\overline{W}) \quad B'_c &= B_c(1-c) \\
I &= B'_c
\end{aligned}
$$

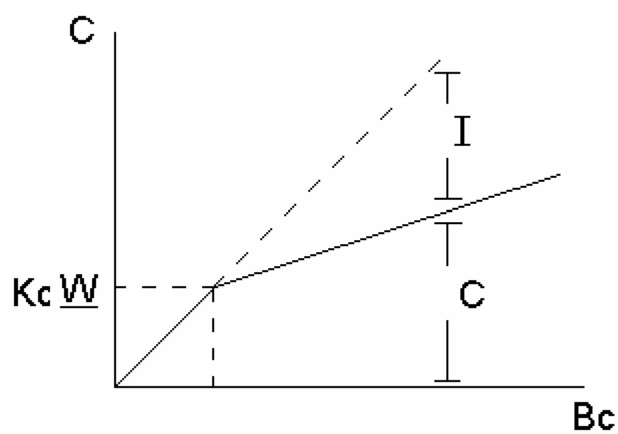



**Figure 3: Consumption and Investment Function**

## Final Wealth

The couple's final wealth is calculated after it has consumed and left an amount to be invested. Final wealth is a function of the couple's human capital and the amount to be invested. Human capital in the final wealth function represents all income generated from earnings and the couple's capacity to generate wealth without material capital. Conversely, material capital represents all income generated as a return to investment. If the couple consumed all of its material capital then its final wealth depends solely on human capital. Final wealth is a Cobb-Douglas function. The exogenous parameters $\rho$ and $\delta$ determine the normal and marginal returns to scale. If $\rho > 1$ and $\delta > 1$ then $W_{BB} > 0$ and $W_{HH} > 0$ respectively, presenting increasing marginal returns. Furthermore, $\rho, \delta > 0$ implies $W_{HB}, W_{BH} > 0$ meaning that there are complementarities between human and material capital. There is no doubt that human and material capital hold positive relations with wealth ($\rho, \delta > 0$). Notwithstanding, the kind of returns to scale and the type of complementarities among assets valid for real economies are yet to be determined. Bowles (2005) discusses evidence where increasing marginal returns and complementarities among assets hold for some settings. The evidence goes along the lines that wealthier individuals face fewer constraints to investment and take riskier strategies providing higher expected returns. It is also shown that returns to education appear to increase with years of schooling. This provides some inclination towards increasing returns to scale in both marginal and human capital. It also seems to be the case that more educated individuals have higher returns to their investments providing some evidence for $W_{BH} > 0$. One may also argue that wealthier individuals have higher returns to human capital due to dependency of human capital factors on wealth, this would provide some evidence for $W_{HB} > 0$.

There is some volatility in the returns to investment and volatility in the returns to human capital related to each individual's personal conditions such as employment environment, particular line of work and luck. This randomness is captured in the model with a shock to the final wealth of the couple. Note that this shock is different from the one related to the inheritance of human capital.

$$\begin{aligned} W_c &: \text{Couple Wealth} \\ H_c &: \text{Couple H} \\ H_c &= H^i + H^j \end{aligned}$$

$$\textbf{if}(B'_c >= 1) \quad W_c = B'^{\rho}_c H^{\delta}_c + \varepsilon_2$$

$$\textbf{else}(B'_c < 1) \quad W_c = H^{\delta}_c + \varepsilon_2$$

## Reproduction

Once the couple has determined its final wealth it proceeds to reproduce. The number of kids each couple has is determined by a linear combination of two factors. The first factor is equal reproduction where every couple has two kids. The second is reproduction dependant on wealth. The second factor gives each couple the same share of kids from total population -which remains constant- as the couple's share of wealth in total wealth, this means that a couple with a share *x* of the total wealth will have a share *x* of the total population as kids. The exogenous parameter $\mu$ allows to have different combinations of the two factors, $\mu$ =1 means that the rich have more kids than the poor, $\mu$ = 0 means that every couple has two kids and $\mu$ = -1 means that the poor have more kids than the rich.

$$\begin{aligned} N &: \text{Total population} \\ n_i &: \text{No. of kids of the i-th couple} \\ \alpha_i &: \frac{W_i^c}{\sum_j^{N/2} W_j^c} \end{aligned}$$

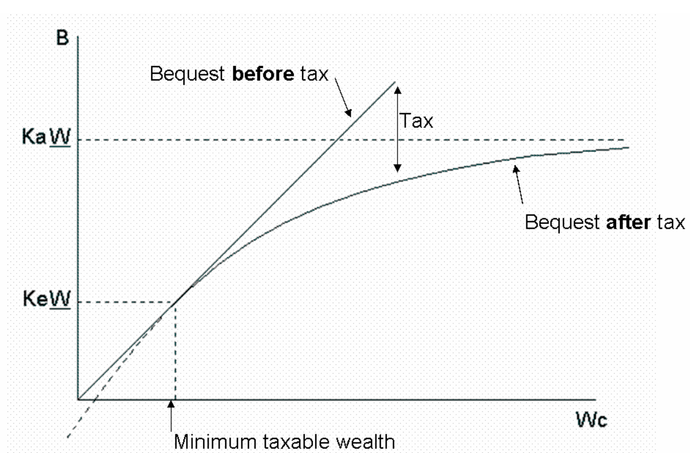
$$n_i = (1-\mu)2 + \mu(\alpha_i N)$$

## Progressive Tax Function

Taxes are modeled as a progressive function that taxes the rich more heavily than the poor. There is no single tax rate, there is a higher rate for each higher level of wealth. There is a minimum taxable wealth, i.e. couples with a lower wealth than this minimum will no be taxed. The minimum is established by the multiplication of the parameter *Ke* and the mean wealth; it means that every wealth lower than *Ke* times the average wealth will not be taxed. Additionally, there is a maximum level of wealth that is allowed to be passed on to children as material bequest. The tax function asymptotically approaches this maximum incrementing the tax rate such that no couple transfers more than the established maximum. The parameter *Ka* multiplied by the mean wealth determines the maximum bequest allowed. Note that *Ka* must be greater or equal than *Ke*. By choosing *Ka* high enough the maximum limit of bequest will no longer be binding and a more realistic situation with no limits to inheritance is possible. Note also that the minimum taxable wealth and the maximum bequest allowed depend on the mean wealth and therefore are endogenously determined and move along the simulation according to the average wealth.

$$
\begin{aligned}
\text{x} &: \text{Parent's wealth} \\
\text{f(x)} &: \text{Bequest as a function of parent's wealth} \\
K_e &: \text{Minimum taxable wealth coefficient} \\
K_a &: \text{Maximum bequest coefficient} \\
\text{f}(k_e\overline{W}) &= K_e\overline{W} \\
\text{f'}(k_e\overline{W}) &= 1 \\
\lim_{x \to \infty} \text{f(x)} &= k_a\overline{W}
\end{aligned}
$$

$$
f(x) = \frac{(x - k_e\overline{W})(k_a\overline{W} - k_e\overline{W})}{(x + k_a\overline{W} - 2 * k_e\overline{W})} + k_e\overline{W}
$$




KaW : Maximum Bequest

**Figure 4: Progressive Tax Function**

## 3. Results

The results consist of Gini coefficients for wealth and human capital, correlation measures of material and human capital within couples and correlation of wealth between parents and children, variance measures of wealth, human and material capital, and finally, distributions of wealth and both types of capital. These measurements are taken for several runs of the simulation for different parameter values that correspond to different situations and to the different effects of each of the driving forces of inequality. The first set of parameters attempt to resemble a modern capitalist society. The results obtained from that run correspond to the benchmark case for the ensuing study of comparative statics.

### Benchmark Case

The probability with which the agents assortate (*r*) is 0.6. The importance of parents' wealth on the inheritance of human capital (*z*) is 0.2. The maximum allowed bequest is 25 times the average wealth (*Ka* = 25) and the minimum taxable wealth is exactly the average wealth (*Ke* = 1). Finally, the importance of parents in the creation of the next generation's human capital ($\lambda$) is 0.6.

The Gini for the distribution of wealth is approximately 0.5 and that of human capital is 0.32. The Gini for wealth is similar to those found for Latin-American countries and even the U.S. The Gini for the distribution of human capital is much lower than the one for wealth as is normally seen in capitalist societies.

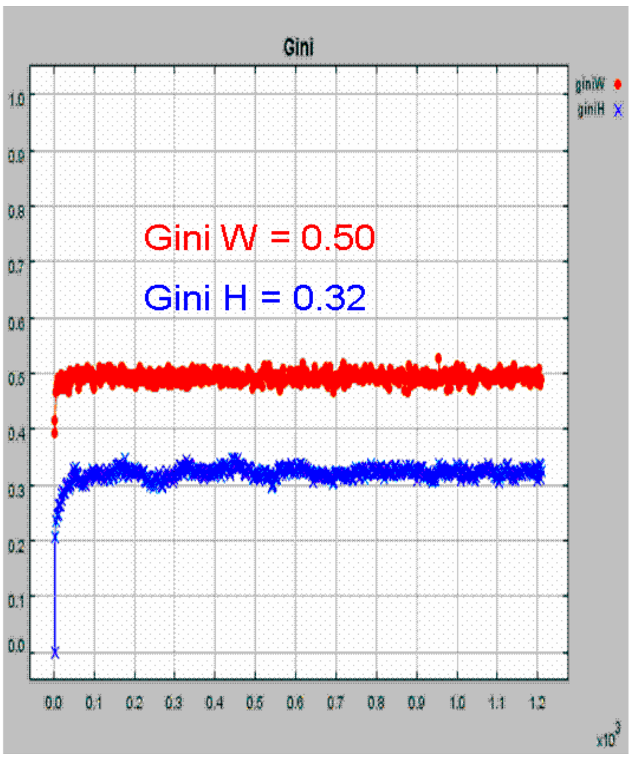



**Figure 5: Benchmark Case Ginis for Wealth and Human Capital**

The correlation of human and material capital within couples is 0.55 and 0.50 respectively. The correlation of wealth between generations is 0.7. These values are hard to contrast specially for the correlation of wealth since the data needed to calculate it relies on the memory and estimations of the people surveyed.

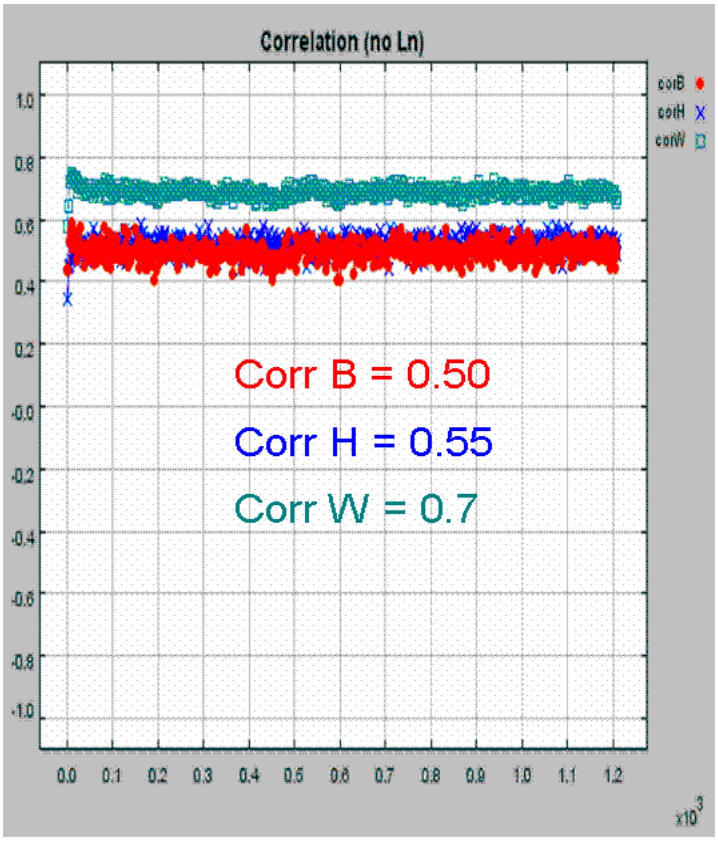



**Figure 6: Benchmark Case Correlations of Wealth, Material and Human Capital**

The distribution of wealth shows that there is a big middle-low class with some very rich people, again similar to what is seen in many capitalist societies. The distribution of material capital resembles that of wealth; this is to be expected since next generation's material capital is the current's generation wealth after taxes. The distribution of human capital is a bell-shaped curve indicating the redistributive effect of taxes.

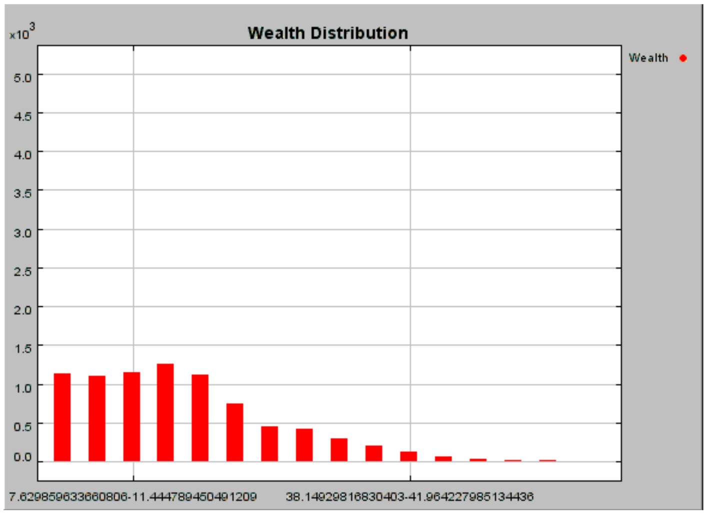



Figure 7: Benchmark Case Wealth Distribution

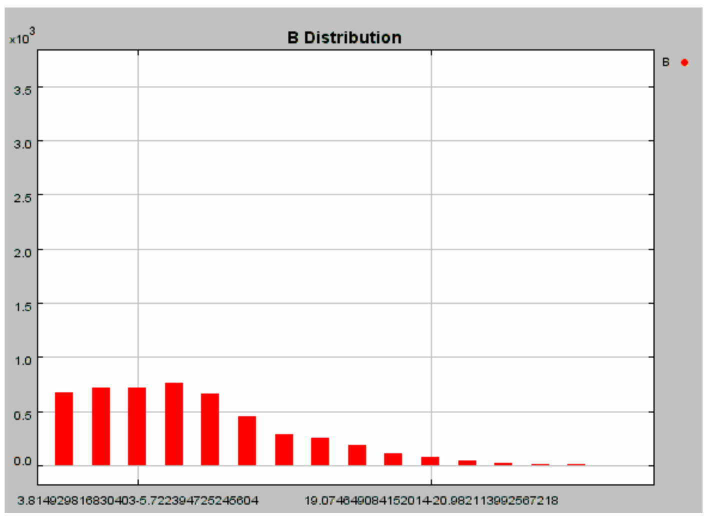



Figure 8: Benchmark Case Material Capital Distribution

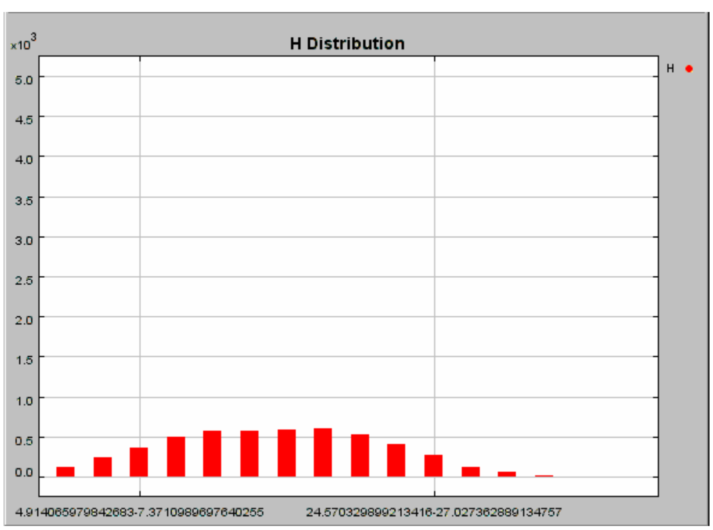



Figure 9: Benchmark Case Human Capital Distribution

We also calculate the probability that children will find themselves in a particular decile of the distribution of wealth given the decile of the parents. In this way we calculate a transition matrix with probabilities of mobility within deciles of the wealth distribution from one generation to the next. Herz (2002) calculates the intergenerational income transition probabilities using PSID data finding that the probability of staying in the lowest decile given that one's parent are in the lowest decile is 0.31, and the probability of staying in the highest decile given that one's parents are in the highest decile is 0.23. The transition matrix for the simulation of the benchmark case gives values of 0.25 and 0.52 respectively. Despite the fact that Hertz's study treats income and not wealth it gives a coarse contrast point to compare the results from the simulation. Although agents in the simulation have a much higher probability of staying rich than what appears to be the case in reality, the transition matrix exhibits the same trend and form as the one obtained by Hertz for the income data.

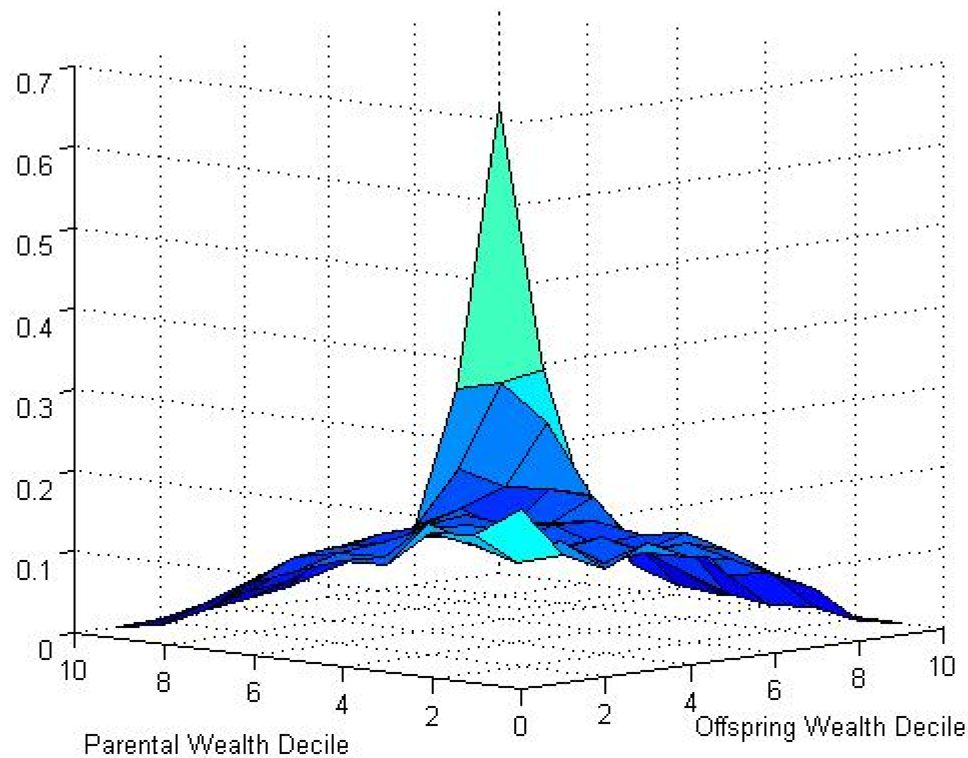



**Figure 10: Benchmark Case Intergenerational Wealth Transition Probabilities**

## Effect of $\delta$

$\delta$ is the exponent of human capital in the final wealth function. The higher this exponent is the lower the Gini for wealth and human capital is but only if $z$ is low, i.e. when parent's wealth is not very important in the formation of human capital. This is a very intuitive result; it is good in terms of inequality to let the generation of wealth depend heavily on the people's human capital but only when the formation of human capital is almost independent of the wealth of parents. Incrementing $\delta$ from 0.3 to 0.7

with a $z$ of 0.2 lowers the Gini for wealth from 0.49 to 0.39 and the Gini for human capital from 0.40 to 0.27, see figure 11.

For a high value of $z$ the converse is true: higher importance of human capital in the generation of wealth is bad in terms of inequality when human capital depends on the wealth of the parents. This is also an intuitive result, when family wealth is crucial in the kids' human capital one would expect a higher importance of human capital in the generation of wealth to generate more inequality. When $\delta$ is incremented from 0.3 to 0.7 with a $z$ of 0.8 the Gini for wealth increases from 0.5 to 0.75, see figure 12.

In conclusion, the effect of the importance of human capital in the generation of wealth depends completely on whether human capital formation depends on the wealth of parents.

## Assortation Probability (*r*)

Stronger positive assortation generates more inequality. Increasing the probability of choosing a partner of the same rank from 0.2 to 0.9 increases the correlation of wealth between parents and children from 0.5 to 0.8, see figure 13. It is noteworthy that changing $r$ with all other parameters being the same as in the benchmark case does not change much the Gini for wealth, nevertheless the increase in inequality is picked up by a higher wealth correlation between parents and offspring.

## Effect of $\mu$

$\mu$ is the parameter that represents the importance of wealth in reproduction. Higher values of $\mu$ generate less inequality, when $\mu$ increases from 0 to 0.4 the Gini for wealth decreases from 0.4 to 0.3 and the Gini for human capital decreases from 0.28 to 0.16, see figure 14. When the rich have more than two kids while the poor have less than two, rich children have to share their parents' wealth with a bigger number of siblings thus generating less inequality in the following generation.

The distributions of material bequests show the same result. For $\mu = 0.4$ the distribution resembles a bell-shaped curve while for $\mu = 0$ the distribution is skewed to the left and has a long right tail, see figure 15. Reproduction inequality generates economic equality.

## Taxes

Taxes are a very powerful equalizing force; the higher the tax the lower the Gini of wealth. Lowering *Ka* from 25 to 5 increases total tax, as a percentage of total wealth, from 1.25% to 5.59% and it reduces the Gini for wealth from 0.39 to 0.34, see figure 16.

It takes a total tax of 18.5% (*Ke* =1, *Ka* = 1.8) to generate the same Gini for wealth that is reached when reproduction depends completely on parents' wealth ($\mu = 1$) and there are no taxes, namely 0.25. In the latter case the Gini for human capital is still lower (0.17 versus 0.25). Taking 18.5% of total wealth as taxes generates the same Gini for wealth and a higher Gini for human capital than having no taxes and letting reproduction depend entirely on wealth. Note that a tax of 18.5% is reached with a very constrained upper limit (no one is allowed to inherit more than 1.8 times the average wealth) and that the social consequences of such a harsh measure are not taken into account.

Even though a relative small tax of 18.5% can have the same equalizing force, under constant returns to scale, than extreme reproduction inequality ($\mu$ = 1) both forces generate different wealth distributions and have different social consequences. A bell-shaped wealth distribution is obtained when a Gini of 0.25 for wealth is reached via extreme reproduction inequality, whereas a right-skewed distribution with a long fat left tail is obtained with a tax of 18.5% (*Kc* = 1.8), see figure 17.

Under increasing returns to scale ($\delta$ = 0.7, $\rho$ = 0.4) extreme reproduction inequality ($\mu$ = 1) and no taxes is a much better equalizing force than taxes for values of *Ka* greater than 1.6. The former force reaches values of 0.25 for the Gini of wealth and a variance of wealth of 0.3, while the latter produces with *Ka* = 1.8 a tax of 99.9%, a higher Gini for wealth of 0.4 and a constantly increasing variance of wealth. For higher values of *Ka* the tax is still 99.9% and the Gini for wealth increases. For values of *Ka* of 1.6 or lower better Ginis for wealth are reached (0.2 and less) with taxes around 25%. Note that reducing the maximum bequest allowed from 1.7 times the mean wealth to 1.6 causes the tax as a percentage of total wealth to fall from 99.9% to 25%, this is due to the explosion on the level of wealth caused by the increasing returns to scale when *Ka* > 1.6.

## Constraints to Investment

Constraints to investment are modeled by the parameter *Kc*, which determines the normal consumption and therefore sets a threshold that separates pure consumers from investors. Every couple with a material capital lower than *Kc* times the mean wealth will have to consume its entire material bequest and is left with nothing to invest.

The effects of changing *Kc* depend on the variable we choose to represent inequality. Lowering *Kc* generates lower Ginis for wealth, see figure 18. For *Kc* = 0.5 or lower everyone is allowed to invest (all couples have more than half the average wealth) and a Gini for wealth very close to zero is reached. For high values of *Kc* (3 and more) everyone is left out of investment and a Gini for wealth of 0.4 is reached. Inequality, in terms of wealth Gini, improves when *Kc* is lowered. This result changes when we look at the correlation of wealth between generations. Correlation of wealth increases for higher values of *Kc* in the range (0, 0.6) where it reaches a maximum of almost one, then it decreases for values of *Kc* greater than 0.6 converging to a correlation of 0.2. In conclusion, when everyone is investing a correlation of wealth of 0.6 is obtained and when no one is investing a correlation of wealth of 0.2 is obtained, with a maximum value of almost one when *Kc* = 0.6, see figure 19.

Investments constraints play an important role generating inequality. Turning off the effect of every other force ($r$ = 0, $\mu$ = 0, no tax) we notice that investment constraints alone can account for a Gini of wealth of 0.3 when *Kc* = 0.7. The distributions of wealth show the separation of pure consumers and investors for different values of *Kc*. For *Kc* = 0.8 a well-behaved bimodal distribution clearly shows the distribution of the two groups. For *Kc* = 0.7 the separation is much more drastic, investors are more separated from consumers and have a long left tail, while the difference among consumers is almost inexistent. When every couple is allowed to invest (*Kc* <= 0.5) there are almost no differences of wealth among agents, whereas when no couple is allowed to invest (*Kc* = 30) wealth exhibits a bell-shaped distribution with very fat tails. See figure 20.

## Volatility, Randomness and Luck

Volatility, randomness and luck are gathered in the shocks to final wealth and to the formation of human capital. Incrementing the magnitude of the shocks generates more inequality. Augmenting shocks from 10 to 100 causes the Gini for wealth to increase from 0.39 to 0.52 and the Gini for human capital from 0.27 to 0.34, see figure 21. This provides useful intuition about the sources of inequality, namely that it can not be entirely explained by structural and distinguishable forces in the economy and society but that part of it is generated by random events that make some do better than others.

To confirm this intuition one can come up with a statistical model that tries to capture all the inequality present in the simulation data. The following model was proposed in Bowles (2005). Let us say that the wealth of the children is dependant on the wealth of their parents in the following manner:

(1) $W_{i+1} = \beta_o + \beta_1 W_i + \varepsilon$

We are interested in some measure of inequality so calculating the variance from (1) gives:

$$\text{var}(W_{i+1}) = \beta_1^2 \text{var}(W_i) + \sigma_\varepsilon^2$$

in the stationary level the variance of wealth from one generation to another will not vary, hence $\sigma_i^2 = \sigma_{i+1}^2$ and we get an expression for the variance of wealth that depends on an explicable source $\beta_1$ and the variance of the noise $\sigma_\varepsilon^2$.

(2) $\sigma_w^2 = \sigma_\varepsilon^2 / (1- \beta_1^2)$

Running a regression with the data from the simulation on equation (1) gives us the value of $\beta_1$. The results obtained are as follows:

$\beta_1 = 0.6444$
$\beta_0 = 5.0715$
$r^2 = 0.5$
p-value = 0.0001

The betas are statistically different from zero and the model explains 50% in the variation in $W_{i+1}$. To check the results obtained from the simulation we contrast them with the statistical analysis. We know that:

$$\rho = \beta_1 \sigma_{wi}^2 / \sigma_{wi+1}^2$$

but in the lung-run $\sigma_{wi}^2 = \sigma_{wi+1}^2$ , hence:

$$\rho = \beta_1$$

and indeed we find that the correlation for wealth between parents and children obtained in the simulation is approximately 0.65, which is practically equal to $\beta_1$. Therefore we can be sure that the results from the simulation are consistent with the statistical analysis. Using the value of $\beta_1$ we can know the relation between the variance of wealth and noise in equation (2):

$$\sigma_w^2 = 1.73\sigma_\varepsilon^2$$

The variance of wealth is 1.73 times that of the noise indicating that 57% of inequality is explained by noise and not by an identifiable source. Note that the simple model in (1) does not have a good prediction capacity ($r^2 = 0.5$) and one could introduce more variables to try to explain better the wealth of the next generation. Nevertheless, the model supports the idea that volatility and randomness play a role in the generation of inequality.

### Effect of $\lambda$

$\lambda$ is the parameter that allows changing the relative importance of society and parents in the generation of human capital. Higher $\lambda$s, i.e. giving more importance to the contribution of parents than that of society in the formation of their children's human capital, generates more inequality. Incrementing $\lambda$ from 0.1 to 0.9 causes the Gini for wealth to increase from 0.3 to 0.5 and the Gini for human capital from 0.19 to 0.32, see figure 22. In terms of inequality, it is better if human capital is provided by society and not by parents.

## 4. Conclusions

The simulation provides a means to analyze the separate and combined effects of the different forces that are believed to drive inequality. The results of the simulation correspond to what one would intuitively think to be true. Another discussion of the results would be redundant thus just a general description of the main message is given.

According to the model it is better to leave the formation of human capital to society than to the parents, more so if the income-generating factors transmitted by the parents depend on their wealth. The main trend seen in modern societies where the sources of income depend more and more on human capital will have a huge negative impact on inequality if the formation of human capital depends heavily on the parents' wealth and is not provided by society. The state can provide high quality education and health care canceling the advantage of private education and health. Good institutions based on meritocracy and equal-opportunity employment can diminish the positive externalities of connections and genetic advantages such as race and looks among others.

Investment constraints are also another important force driving inequality. The poor have limited access to credit and due to their low income it is difficult for them to accumulate the necessary savings for investment. There is also the complementary effect, which describes the higher returns to investments done by individuals with more human capital, i.e. with more and better information and a better capacity to harness it. The role of society and the state removing the constraints to investment is limited. More equal human capital and access to pertinent information can help, but the principal obstacle preventing investment is the low saving capacity of the worse-off. Despite the fact that more equal human capital does not directly remove the constraints to investment it can indirectly improve and make accessible to everyone the opportunities to generate a higher income that permits saving.

According to the simulation, stronger positive assortation generates more inequality, but compared to the other forces its impact is not decisive on the stationary level of inequality. People may continue to enter long term relationships only with those they consider similar, i.e. same status, wealth, human capital and race to name a few, without that having an important negative impact on inequality. Positive assortation can even be neglected if human capital is transmitted by society and the constraints to investment are not very strong.

The simulation clearly shows that reproduction inequality generates economic equality. When the number of kids a couple has depends on its wealth rich kids will have to share their parents' wealth with more siblings and the gap between them and the poor will shorten. Nonetheless, if human capital depends heavily on the parents' genetic and cultural transmission reproductive inequality will not have a big positive impact on inequality.

Taxes are another very equalizing force. Taxes are taken from the parents' wealth right before they pass it to their children and they are returned to society as human capital formation. If taxes are spent in this way then it is better to have human capital as the main source of wealth. Taxes in the simulation are strongly progressive and impose an upper limit to material bequests. It should be bared in mind that generally taxes are not so heavily progressive nor they impose an upper limit on wealth or bequest and that the social consequences of such a tax are considerable and not taken into account.

Finally, the simulation also suggests that not all the forces driving inequality are identifiable and are due to economic or social causes. Volatility, randomness and luck also generate inequality. Particular events that happen to some individuals and not to others may set them on different paths and affect how they do in life. The state can buffer shocks with a uniform presence in the society, e.g. institutions of same quality and equal enforcement in the entire nation. Some shocks as the returns to investment are harder to centrally buffer.

There is a chance for more equal societies if the most important source of wealth is human capital –as seems to be the current trend- and human capital formation is left to society and not to parents, i.e. states that provide high quality and widespread institutions in an economy that is highly dependant on human capital will lead to equality.

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## Figures

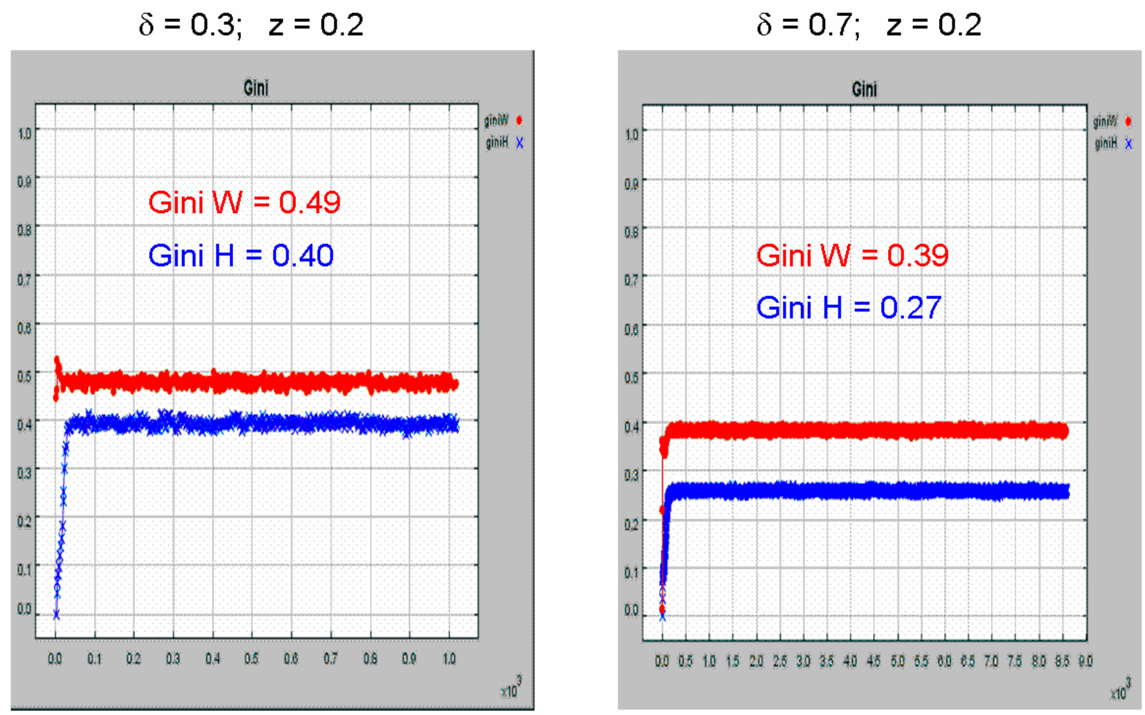


**Figure 11: Gini, Effect of δ for Low Z**

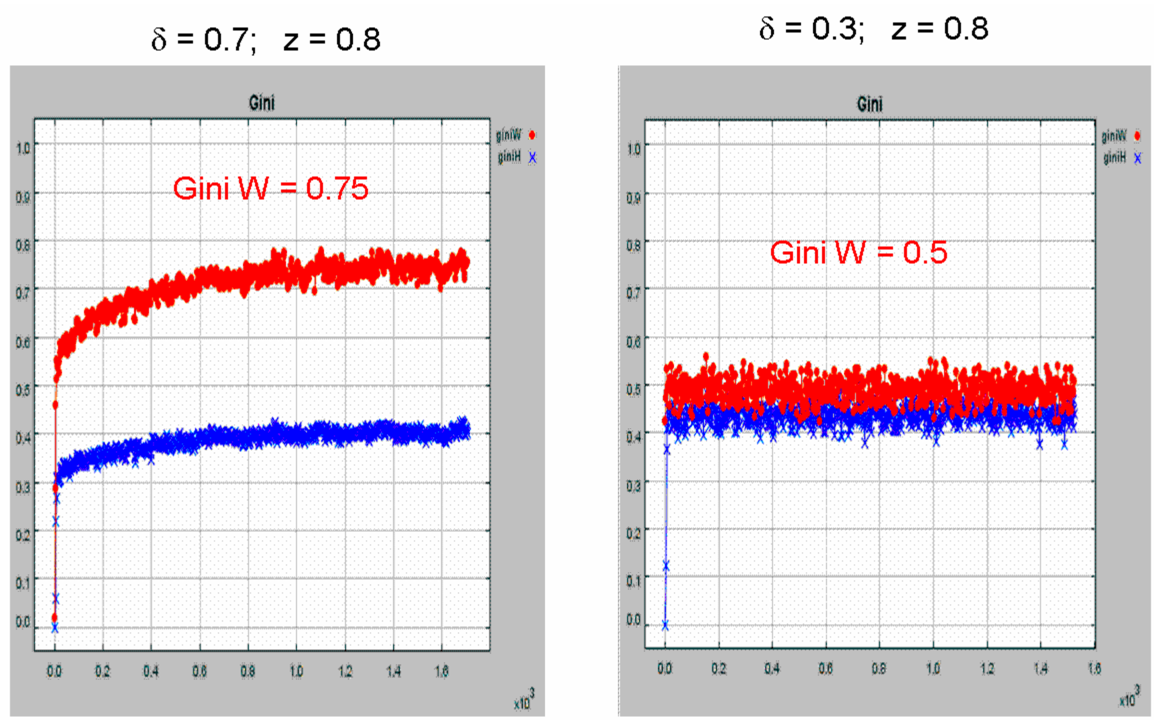


**Figure 12: Gini, Effect of δ for Hi Z**

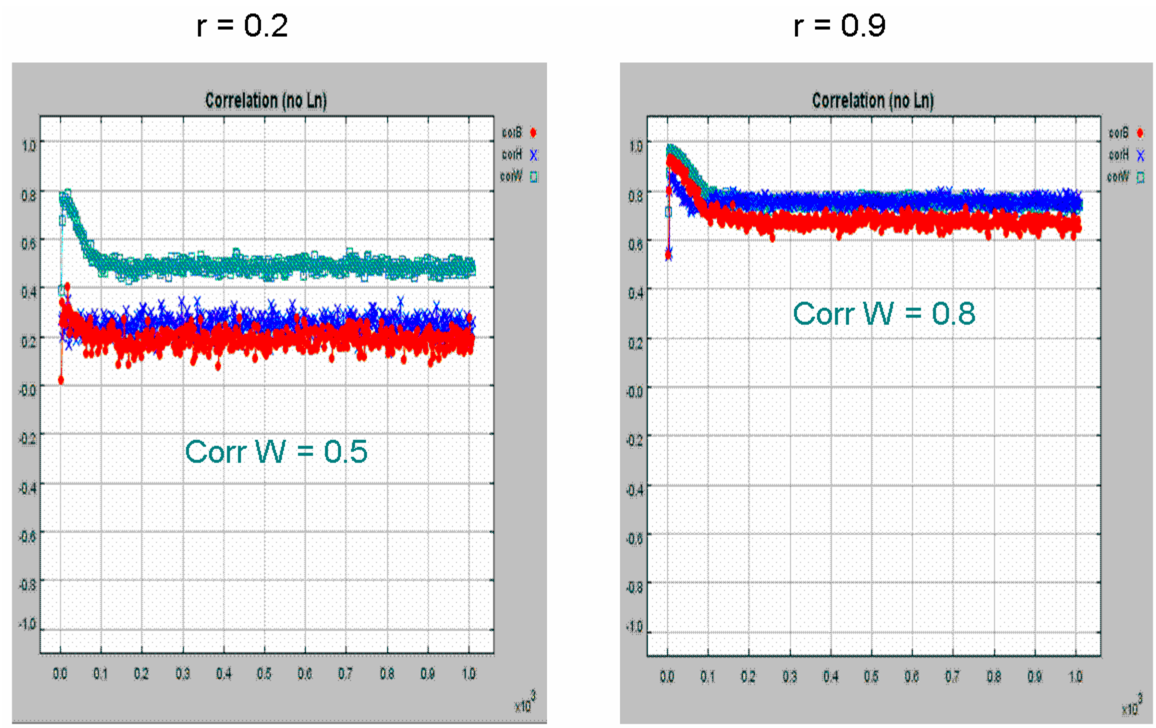


Figure 13: Correlations, Positive Assortation (*r*)

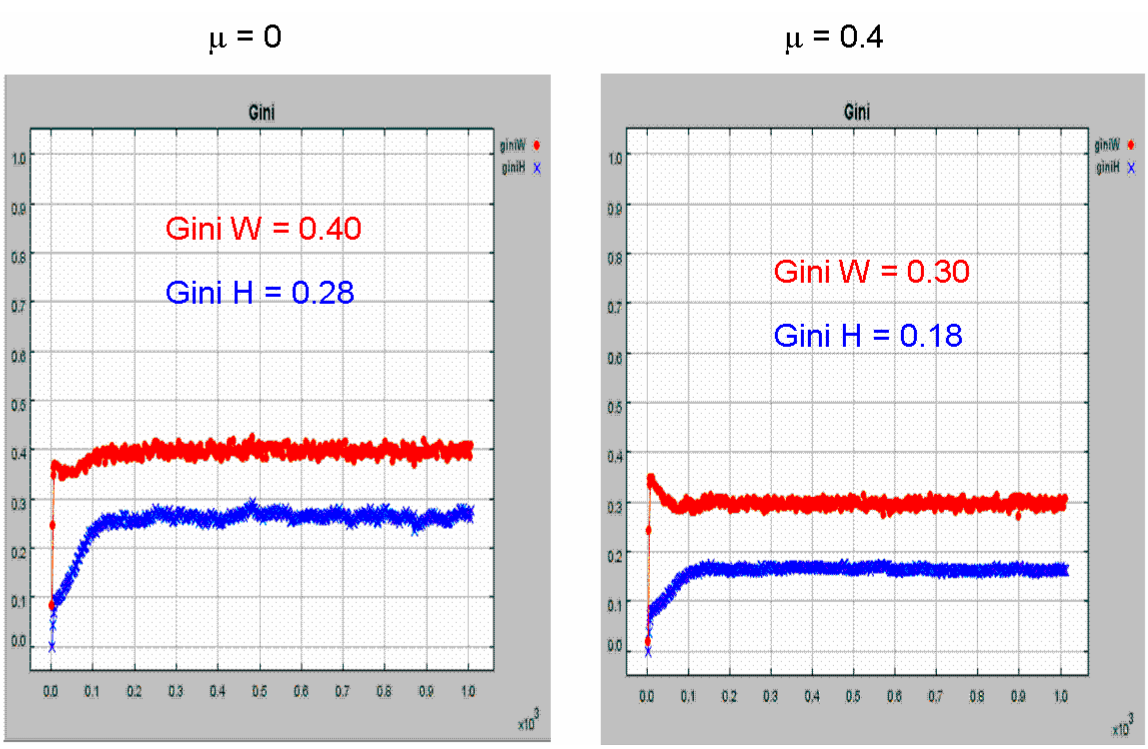


Figure 14: Ginis, Effect of μ

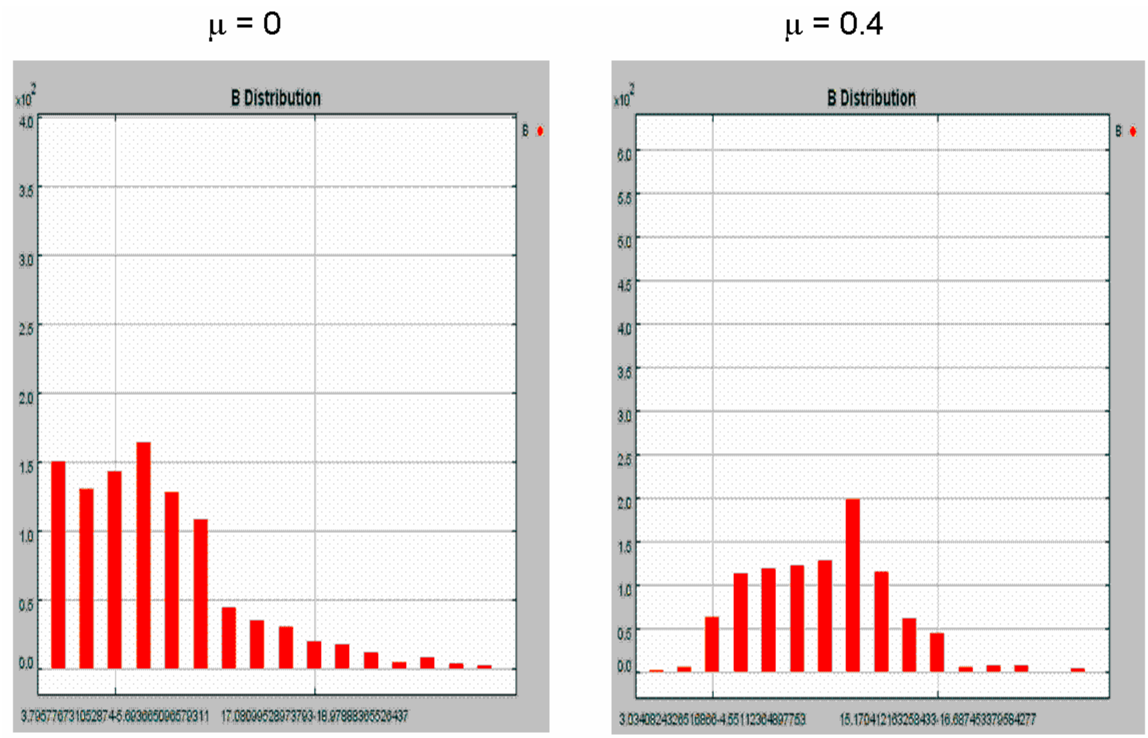


**Figure 15: Bequest Distribution, Effect of μ**

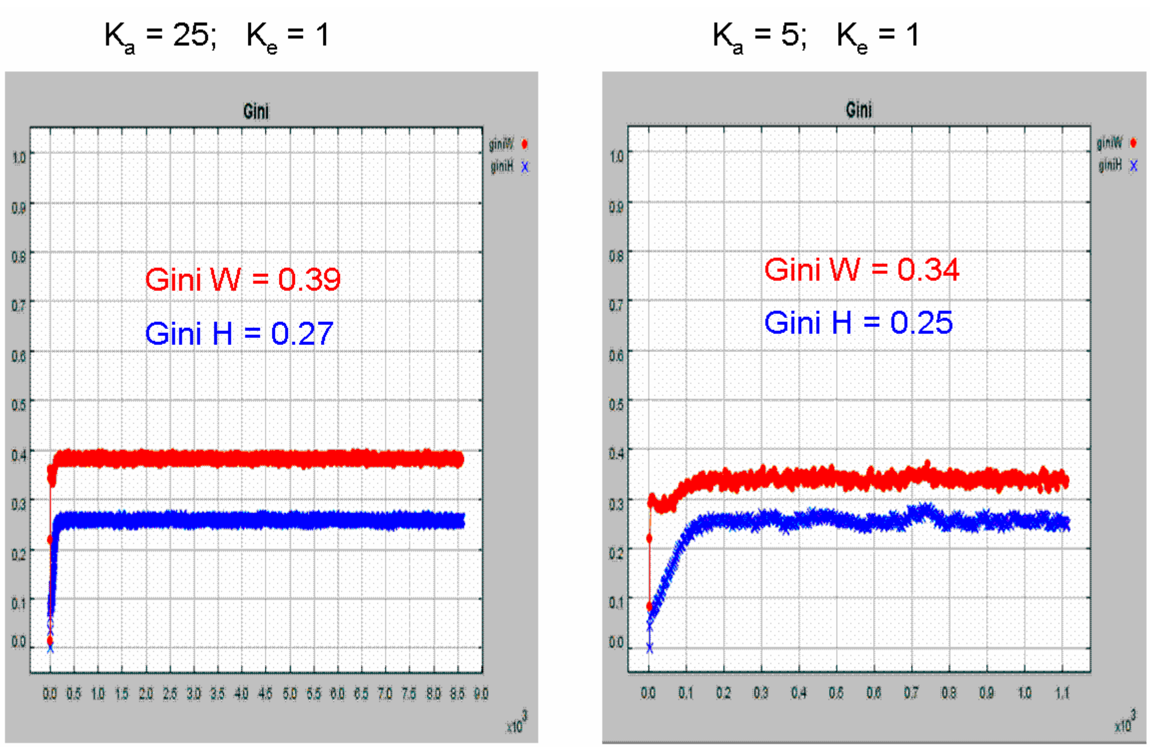


**Figure 16: Ginis, Taxes**

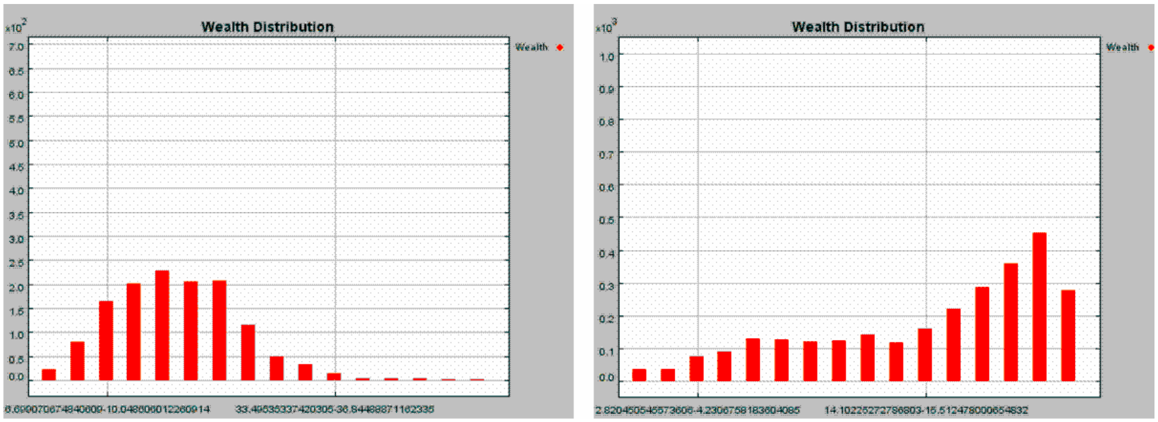


**Figure 17: Wealth Distributions, Taxes.** The graph to the left shows a more egalitarian distribution of wealth when a Gini of 0.25 is reached via reproduction inequality ($\mu = 1$) with zero tax and not via higher taxes and reproduction equality ($\mu = 0$).

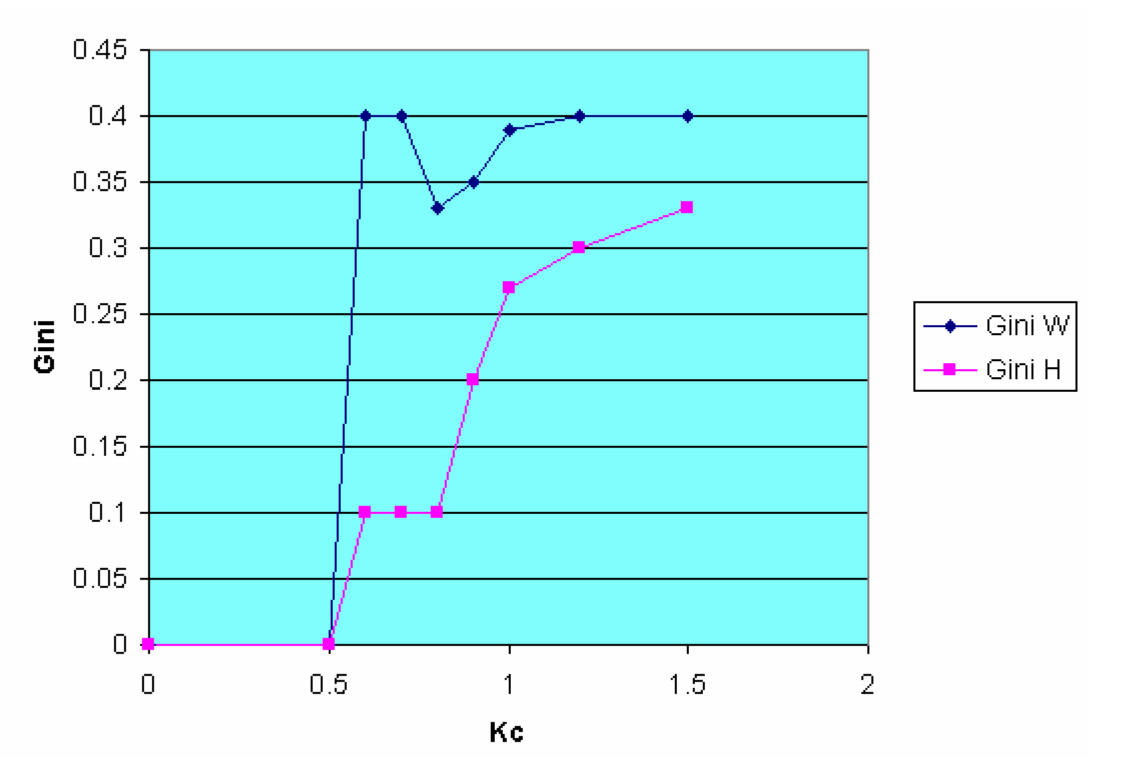


**Figure 18: Ginis, Investment Constraints (*Kc*)**

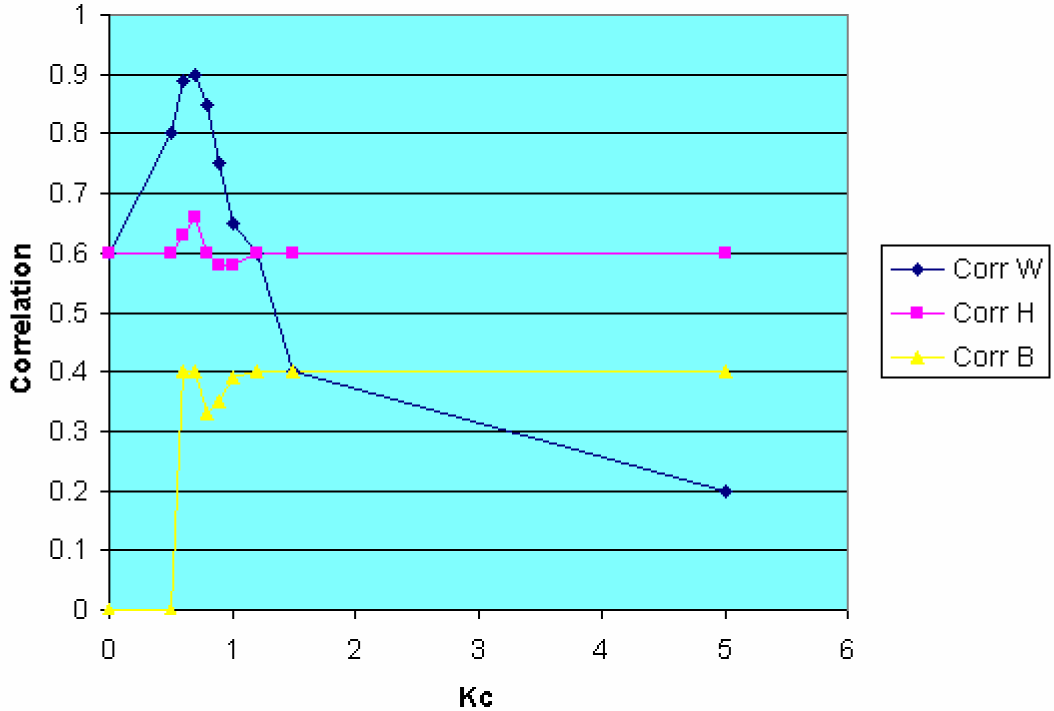


**Figure 19: Correlations, Investment Constraints (*Kc*)**

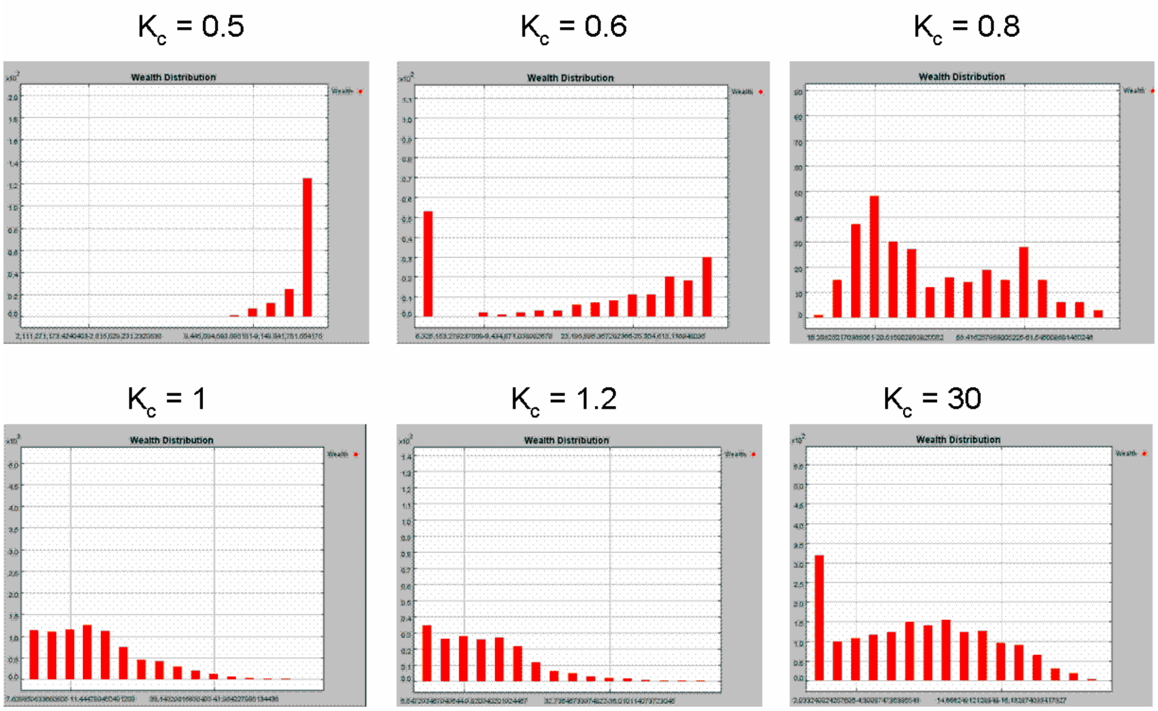


**Figure 20: Wealth Distributions, Investment Constraints (*Kc*)**

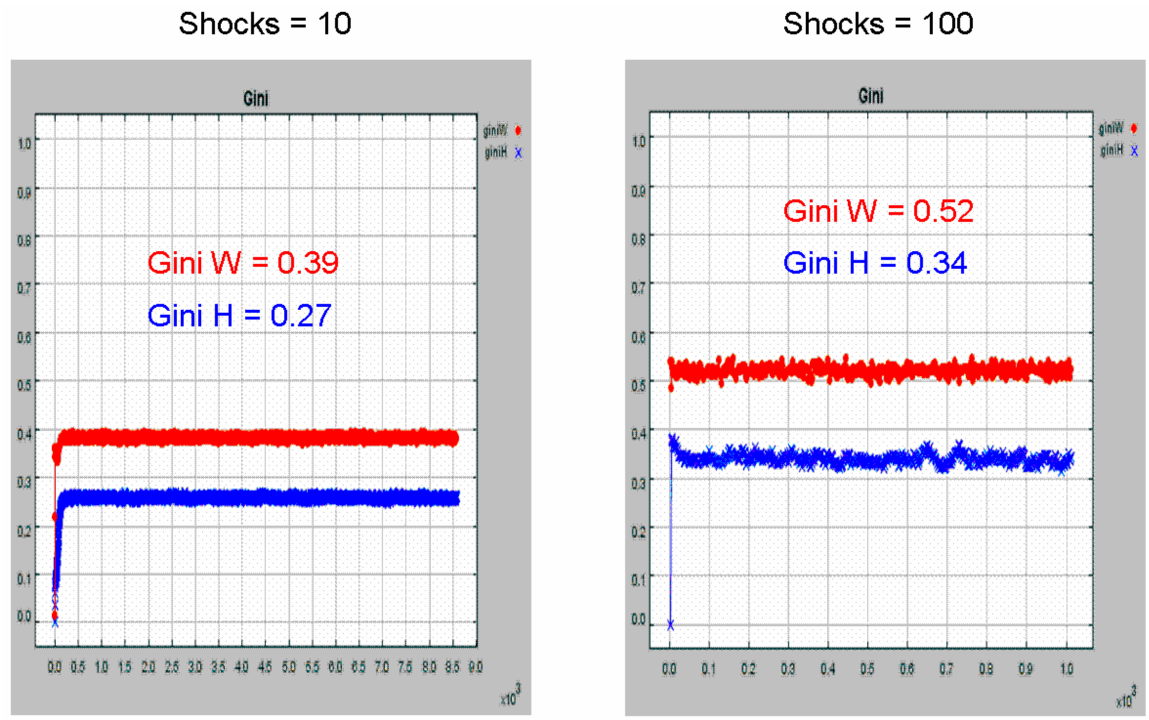


**Figure 21: Ginis, Effect of Shocks**

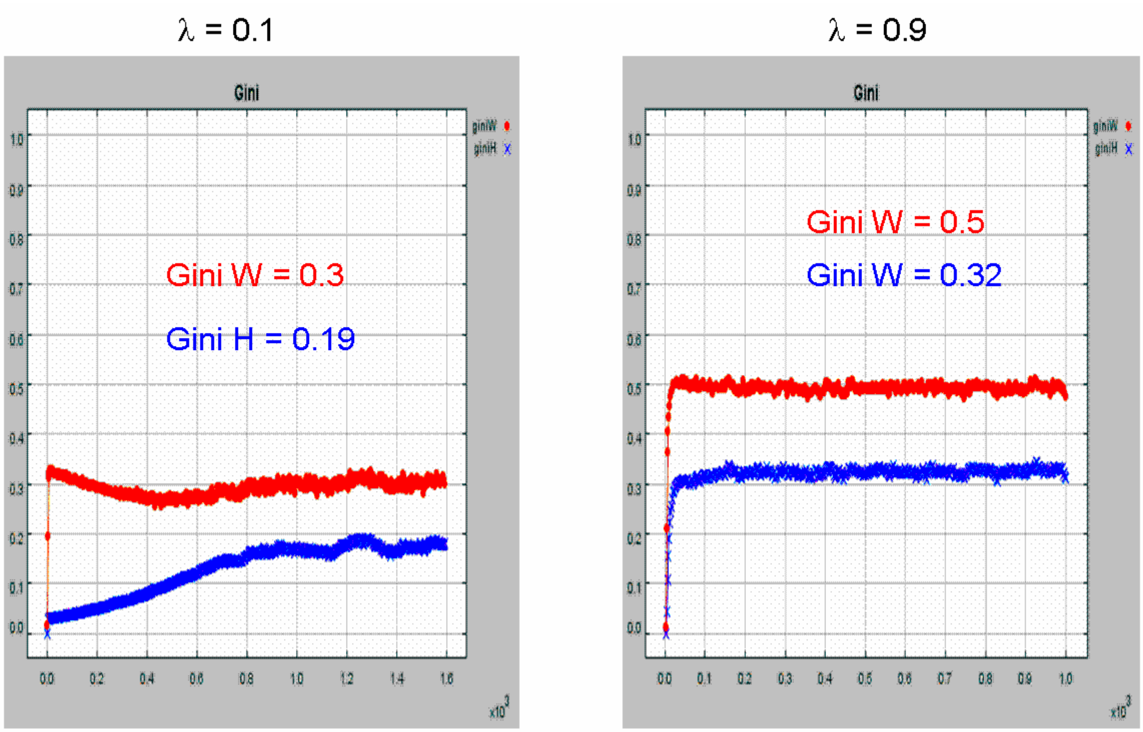


**Figure 22: Ginis, Effect of λ**