Scaling, Aggregation, and Energy Consumption in the UK

Kevin Carlson

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1 Introduction

Much of the history of human civilization is reducible to the history of human cities, from the first stirrings in Sumer and Babylon to the glory of Rome, London, and Beijing at their imperial heights. No one has made a great advance in any human endeavor cut off from these centers of human activity. Even those great thinkers who have been noted for their reclusiveness are not exempt: Spinoza's, Salinger's, and Perelman's solitary adult genius cannot be separated from their youthful immersion in Amsterdam, Manhattan, and Leningrad. These were not idle backdrops for human deeds-rather, the greatest cities attain a vibrant character, an apparent life second to no figure of history or legend. So it is natural that those thinking about cities are drawn to parallels between them and the biological life forms we can observe so much more easily.

Exciting progress has been made on understanding the latter, familiar, sort of life in the past two decades from the perspective of self-similar scaling [7]. It appears that animal bodies of a given bäuplan maximize the efficiency of their metabolism, which produces economies of scale in energy consumption. These align, empirically, with predictions over a much broader domain than has characterized theoretical biology to this point. With predictions of continuing rapid urbanization through the 21st century, alongside those of increasingly destructive impacts from climate change, it became a crucial question whether cities are the consumption-minimizing larger cousins of blue whales and sequoias.

In 2005, high-quality, high-resolution data on carbon emissions and energy consumption was not to be had, so the team gathered to pursue a scale-free understanding of cities focused on more tractable questions. They made a string of discoveries about physical infrastructure and intangible social activity which pointed to an altogether different picture than had been drawn in the biological case [2] [1] [4] [3] [5]. In short, many variables actually showed the inverse of an economy of scale, in which larger populations implied more intensive consumption or production. This left wide questions open regarding the processes that would produce behavior in such contrast to the norm of the biosphere.

2 British Carbon Emissions

Since 2005, the British government has been publishing national statistics on district-by-district energy consumption in each of several sectors. Meanwhile, the European Union has been refining its attempt at a metropolitan area definition in its Urban Audit, which has produced district-level boundary definitions for 65 LUZs, or Larger Urban Zones. This is a coarse definition for such a fine-grained structure as a city, but the best yet developed for the UK and the only one allowing comparisons between EU countries. The product of this aggregation is mapped in Figure 1, where the difficulties arising from the close packing of urban areas in the UK is apparent. Altogether, it appears that energy consumption is likely linear across all sectors.

As precedent has suggested, domestic energy variables looked quite linear in the British data. As an example, Figure 2 shows domestic electricity consumption, which is the most accurate component of the dataset in being constructed from individual meter readings. The β is very near 1, indicating on a log-log plot trivial or linear scaling. Transport variables looked to have lower slopes, as Figure 3 demonstrates. Yet in contrast to American data, we can only rule out a linear relationship, indicating constant per capita consumption over population, at a very marginal significance level. This is somewhat counterintuitive, implying that residents of larger cities make up for the gain of being packed closer together by either living farther from their destinations or traveling more often. However, that explanation makes considerable sense in the light of an economic perspective on social variables in the city, where individual agents are striving to more or less maximize their consumption at any given time. Since income is also higher in larger cities, London residents often have more money available to move around than those in smaller areas.

Finally, total energy consumption looked more sublinear yet, but this is explained by the strong outliers Chester and Falkirk on the lower end of population. These two cities happen to be exactly those that host oil refineries, so their industrial consumption levels greatly exceed expectations. This pattern of energy consumption, in which larger cities seem to benefit from limited or no economies of scale, is in stark contrast to the greedy pursuit of efficiency that has characterized biological evolution. Since the infrastructure that supports energy use *does* scale sublinearly, these results call for a characterization of the forces that cause residents of large cities to waste energy relative to their neighbors.

3 The Disaggregation Problem

One observation which impinges on the energy consumption results arises from the ongoing effort to define the city. In datasets from multiple nations a pattern is observed that, when regression coefficients are calculated for a given energy variable at a lower level of aggregation, the resulting β tends to be lower than the corresponding one at the city level. If this pattern cannot be easily explained, it

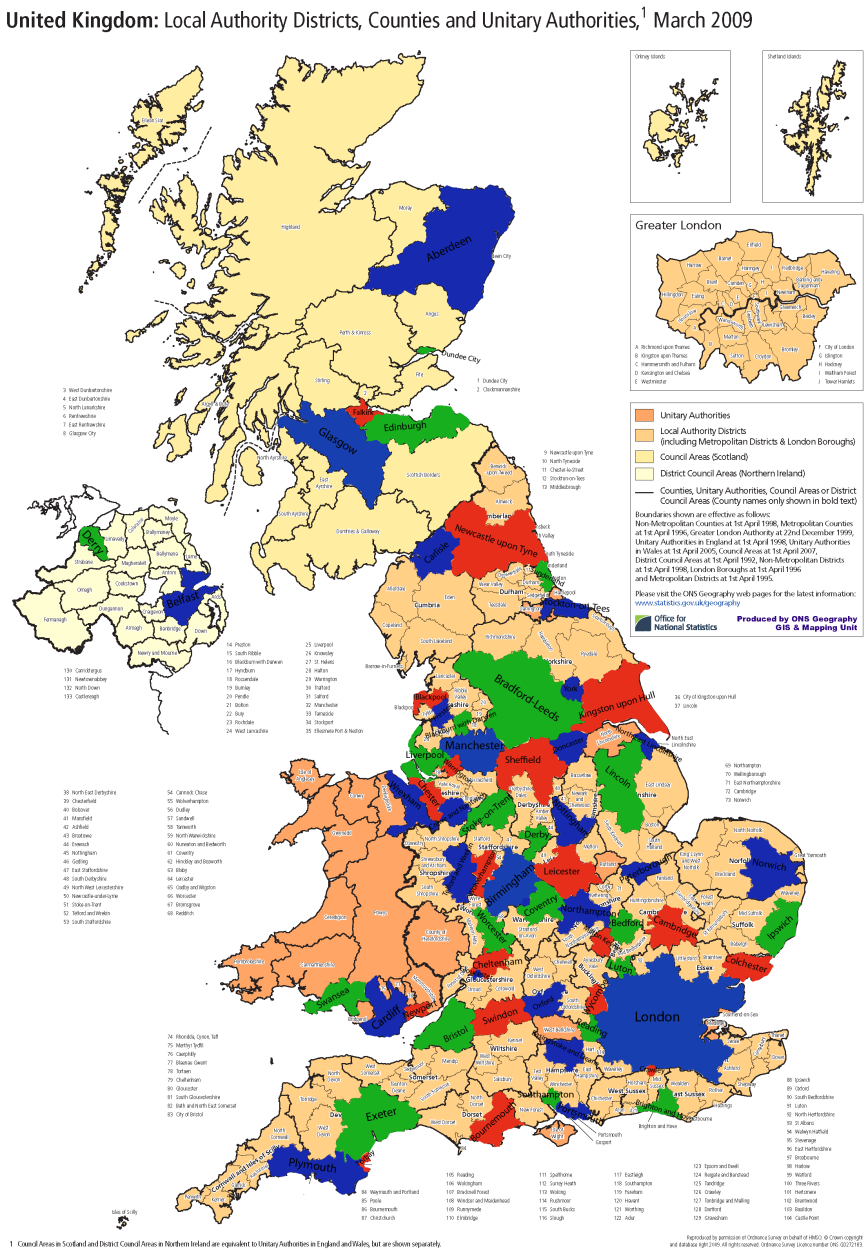

Figure 1: The red, green, and blue colored areas indicate the coverage of each LUZ.

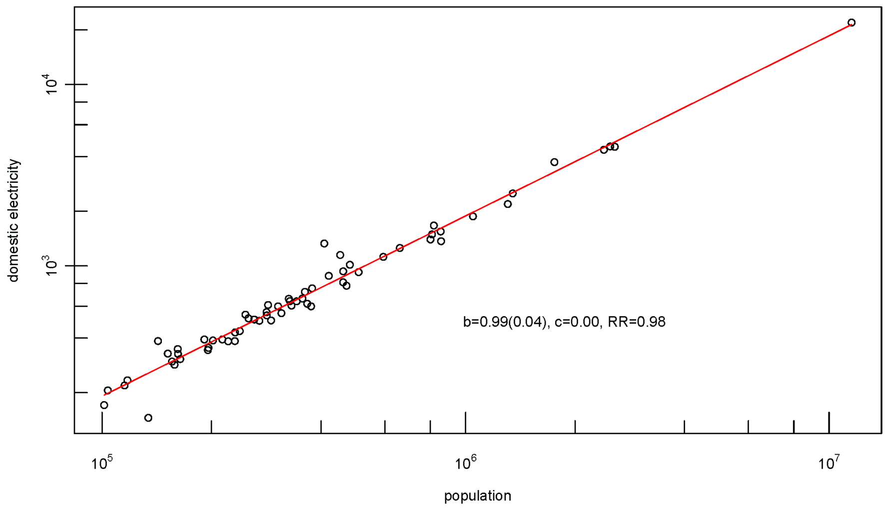

Figure 2: Domestic electricity consumption by population.

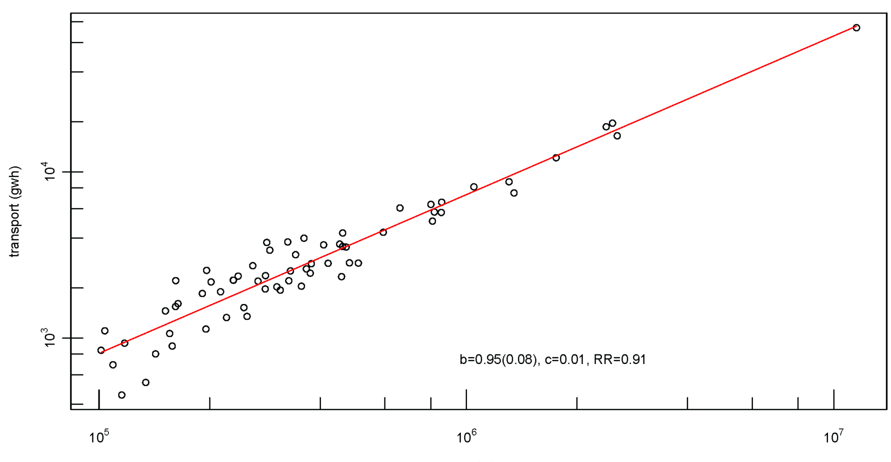

Figure 3: Transport energy consumption, including private road use and freight.

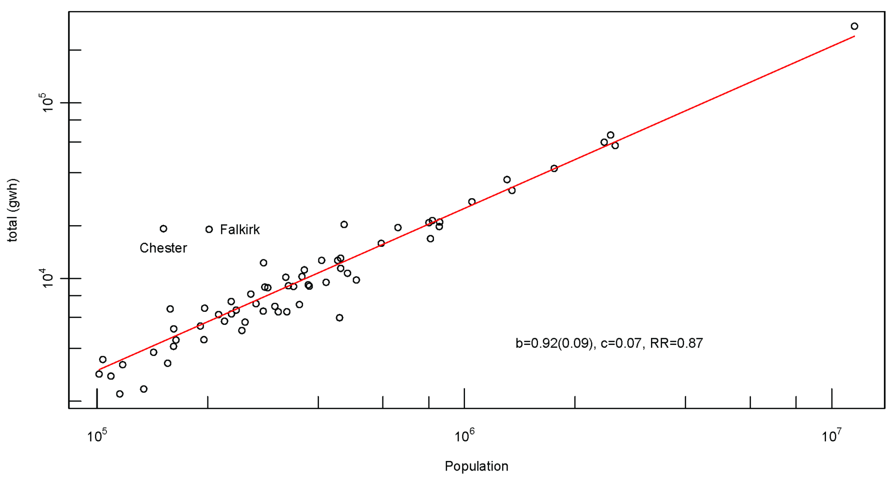

Figure 4: Total energy consumption, including industrial and commercial-sector.

both requires and may inform a rigorous basis for defining the extent of a city, or all manner of urban laws can be discovered simply by using inconsistent levels of description in turn. In some cases, this disaggregation problem is actually a chimaera.

The following derivation demonstrates conditions under which the aggregate variable Y's scaling exponent β is approximated by the Pearson correlation coefficient ρ between Y and population, N.

Let $y = Y/N$. Then in our context y is the per capita value of Y and we have

$$\begin{aligned} Y &\equiv yN \\ \ln Y &= \ln y + \ln N \\ \mathrm{Var}(\ln Y) &= \mathrm{Var}(\ln N) + \mathrm{Var}(\ln N) + 2\,\mathrm{Cov}(\ln y, \ln N) \end{aligned}$$

From here on we will use the variables Y, y, N to refer to their logarithms. The OLS slope of Y ($\ln Y$) on N is given by

$$\beta = \rho_{(Y,N)}\sigma_Y/\sigma_N$$
$$= \quad \rho_{(Y,N)}\xi$$

where

$$\xi = \sqrt{\frac{\mathrm{Var}(y) + \mathrm{Var}(N) + 2\,\mathrm{Cov}(y, N)}{\mathrm{Var}(N)}}$$

If we assume $\mathrm{Var}(y) + 2\,\mathrm{Cov}(y, N) \ll \mathrm{Var}(N)$, we have

$$\xi \quad \approx \quad 1$$

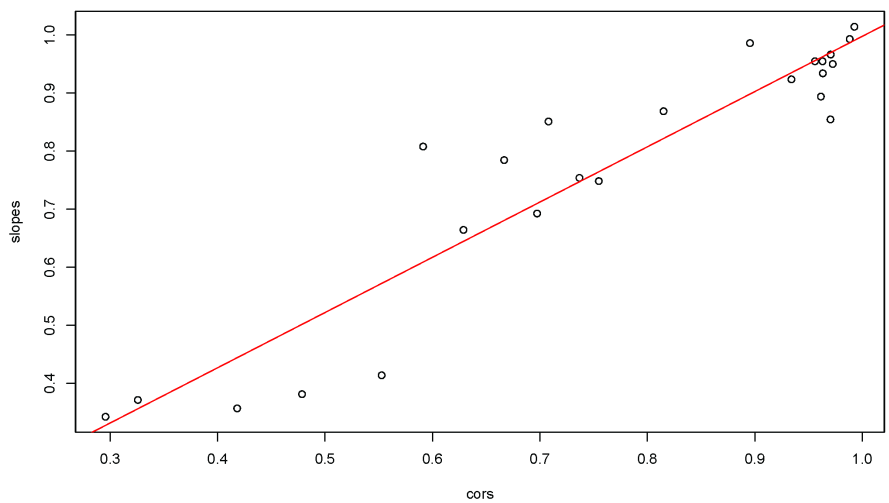

Figure 5: The x-axis shows the correlation between population and 24 different measures of energy consumption; the y-axis shows the slopes of the corresponding regressions, which track very closely.

$$\beta \quad \approx \quad \rho$$

The above assumption is rather strong, and in fact American county data does not observe the derived approximation. Further, for superlinear variables the covariance term will tend to be positive, and the relation will almost never hold. However, the British data does empirically see a strong trend to lower ρ at the LAU than the LUZ level, coupled with $\mathrm{Var}(y)$ and $\mathrm{Cov}(y, N)$ terms that are both small relative to $\mathrm{Var}(N)$ and opposite in sign. Thus for this data β is almost wholly determined by ρ. In Figure 5, we see evidence of this for 24 fits, which are of the 8 most precise energy variables on population at three different levels of aggregation, in order of size:

- LUZ
- LAUs which are part of some LUZ
- LAUs which are part of London

In such situations, we can benefit from a line-fitting technique that's not sensitive to such statistical artifacts. The technique recommended for situations in which only a comparison between two slopes is desired, and not the best ability to make predictions from them, is the Standardized Major Axis (SMA) regression [6], which calculates the slope as

$$\beta_{SMA} = \sigma_Y / \sigma_N$$

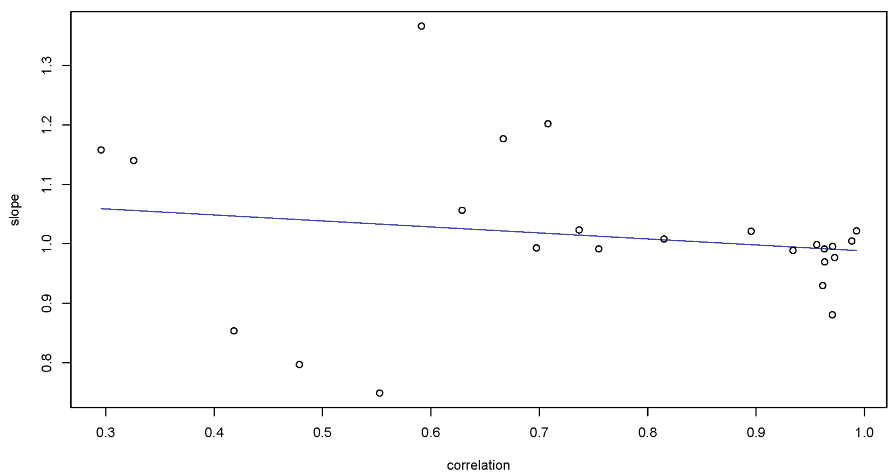

Figure 6: The lack of correlation between correlation and slope for 24 SMA fits.

This is simply the OLS slope divided by the correlation term, and when the above 24 regressions are repeated using SMA, the disaggregation trend disappears (Figure 6).

4 Discussion

Future work includes a more broad understanding of the disaggregation problem in countries in which the above treatment does not suffice. There is also a body of data about physical road infrastructure that will inform a model of transport energy consumption based on the push and pull between the efficiency of the road network and the individual tastes of city residents. The moral we can take from the story of this quite-messy data is that we must have stronger models to make inferences about the forms of the relationships we're interested in. Barring a huge increase in the detail of information on their urban areas countries are willing to support-and to release-this is the only clear way forward in developing an understanding of the physical, biological, social structures in which we live.

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