The Evolution of Economic Inequality and Political Hierarchy on Social Networks

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Abstract Bowles (2009) proposed that the emergence of political hierarchy and economic inequality during the early Holocene might be explained by a progression of three equilibrium network structures derived from the economic and social conditions of the time. Bowles (2009) and Kets et al. (2011) determined the maximal degree of inequality that can persist on a network where coalitions may withdraw from the network, and the set of feasible coalitions is determined by network structure. Here I illustrate their models by studying the maximum inequality sustainable under costs and network decay conditions that may have existed during the Pleistocene and the Holocene, as well as the associated network efficiency, welfare, and levels of inequality. Further, I extend their work by studying the evolution of 4-node network structures subject to 2 dynamics: individually motivated endogenous network evolution and between network competition. Each network evolves according to the pairwise stability rule. Nodes may be shortsighted or far-sighted; the latter anticipate the effect of their actions on bargaining power and the network and hence foresee their post bargaining payoffs while the former consider only the primary payoffs from the network structure. In addition to within network updating, conflicts take place between networks, in which the networks with higher total utility win and replace the losers with a replica of their network structures. The model results show that homogeneous networks converge to the efficient outcomes when the agents on the network are rewarded with anticipated payoffs while the heterogeneous population networks produces diverse distribution of network structure types. We find that within network transitions and between network competitions work in opposite directions because of the concavity of utility function. Finally, the evolution of economic and political hierarchy not only depends on economic and political structures, but also the accessibility of network structure among the possible network structure set.

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1 Introduction

Whether it is for exchange of information, trading of material goods, or reciprocation of favors in times of need, people form social linkages with other people by chance and through deliberation. Such behavior make up complicated and evolving social networks of distinct patterns across cultures and times.

Bowles (2009) proposed that the emergence of political hierarchy and economic inequality during the early Holocene might be explained by a progression of three equilibrium network structures derived from the economic and social conditions of the time. Bowles (2009) and Kets et al. (2011) determined the maximal degree of inequality that can persist on a network where coalitions may withdraw from the network, and the set of feasible coalitions is determined by network structure.

In this paper, we address these questions through theoretical analysis and computational simulation of the distribution of payoffs and utility on a 4-node network. The paper contributes to the analysis of inequality on social networks with non-zero indirect link value. The paper is also an initial attempt to simulate social network formation taking account of political hierarchy and bargaining power on social networks. The simulation model take into consideration both the between group competitions and endogenous within group transitions in the process.

In section 2, I conduct static analysis on the maximum inequality that is sustainable under costs and network decay conditions that may have existed during the late Pleistocene and the early Holocene. In section 3, I study the evolution of 4-agent network structures subject to 2 dynamics: individually motivated endogenous network evolution and between network competition.

2 Economic inequality and political hierarchy on networks

Players are located on a network. A network with N nodes and adjacency matrix g is defined by (N, g). g is an $N \times N$ matrix with $g_{ij} = 1$ indicating that i is connected to j. In such a case, i and j are *adjacent* to each other. The degree of a certain node i is its *degree*, denoted by $d_i(g)$. The *distance* s between two nodes is the length of the shortest path between i and j in g. We say the distance between i and j is infinite when there does not exist a path between the two nodes.

Each node represents a household. The *substance* that is transferred on the network mainly consist of food, right of visitation and information. The payoff π from connections with other households decays at the exponential rate δ with each step of distance away beyond one's immediate neighbor, from which one receives a normalized payoff 1. For example, the payoff from being connected to a household that is 3-distance away is δ^2. Furthermore, maintaining a direct link with another household incurs a cost of c. Then the payoff for household i from its social linkages in the social network (N, g) is

$$\pi_i(g) = \sum_{s=1}^{N-1} d_i^s(g)\delta^{s-1} - d_i^1(g)c,\ d_i^s(g) \in \mathbb{N}, s \in \mathbb{N}. \tag{1}$$

where d_i^s denotes the number of household that is $s-$distance away from household i.

We further assume that the utility of a household u_i from its social linkages is increasing with the payoff and concave in its shape, representing diminishing returns. For the static analysis, we define the utility function for household i with payoff π_i to be

$$u_i(g) = \ln(\pi_i(g) + 1). \tag{2}$$

The total payoff Π - *efficiency* - and the total utility U - *welfare* of the social network (N, g) are

$$\Pi(g) = \sum_{i=1}^{N} \pi_i(g). \tag{3}$$

$$U(g) = \sum_{i=1}^{N} u_i(g). \tag{4}$$

As in Bowles (2009), we distinguish between *economic structure* and *political structure* of a given network. The economic structure is defined by the flow of substances over a network. The political structure is given by the economic network and the set of rules for assignment of bargaining power and how nodes can collectively deviate and form deviation networks.

We call the payoffs and utility that derived from the economic network structure in the absence of bargaining power the *primary* payoff and utility of the network. The nodes on the network are then allowed to transfer and redistribute their payoffs in accord to the economic structure and political structure of the network. To examine the maximal degree of economic inequality that can be sustained on a network, we assume the essential intermediaries between nodes that assume the strictly highest centrality have bargaining power, i.e., they may credibly commit to a "take-it-or-break-it" demand for a transfer from the nodes among which they are essential (Bowles, 2009). If the demand is refused, the central node would sever the link. The maximum level of the transfer that an essential intermediary can demand depends on the deviation network of its peripheries, i.e., the peripheral nodes can singly or collectively deviate from the essential intermediary and form coalition that gives competitive or better payoffs and utility to what they get conceding to the essential intermediary's demand. We assume the essential intermediary is aware of the bottom line of its peripheries and never demands more than what she can without inducing the deviation. Finally, as in Bowles (2009) and Kets et al. (2011), the peripheries can only jointly deviate if they are all within k distance with each other. We explore the changes in the distribution of payoffs and utility when we vary k.

We resort to the *Gini coefficient* to indicate the degree of inequality in both payoffs and individual utility in our network analysis. The Gini coefficient is most easily calculated from unordered size data as the "relative mean difference," i.e., the mean of the difference between every possible pair of individuals i, j in a populatiaon of size n, divided by the mean size μ (Dixon et al., 1987; Damgaard and Weiner, 2000).

$$G = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j|}{n^2 \mu}, \ i, j \in \{1, 2, ...n\} \tag{5}$$

We use the *adjusted Gini coefficient* $G\frac{n}{n-1}$, ranging from 0 where every node receives the same amount of payoffs and utility to 1, where all but one member receives zero payoffs or utlity, to gauge the level of inequality of both payoffs (G_π)and individual utility (G_u) on a network.

2.1 Complete networks

Bowles (2009) hypothesizes that under late Pleistocene conditions (prior to about 12,000 years ago), the economic network tends to be dense. Thus the prototypical Pleistocene economic network is complete, as shown in figure 1a. For concrete analysis, we assume $\delta_P = 0.1$ and $c = 0.1$ based on the facts of the perishability of the substance and the low cost of maintaining social links in the Pleistocene epoch.

All nodes on the Pleistocene complete economic network structure have the same betweenness. As a result, none of them is an essential intermediary, and the egalitarian distribution is the *only* theoretically feasible economic structure that conforms to our model assumption. In a 3-node Pleistocene complete economic network, each node is connected to the other 3 nodes so all nodes share the egalitarian payoffs and utility, as shown in figure 1a. $\pi_i(g_{1a}) = 3 - 3c$; $u_i(g_{1a}) = \ln(\pi_i(g_{1a}) + 1), \ i = 1, 2, 3, 4$. We assume that $\delta_P = 0.1$ and $c = 0.1$, which give us $\pi_i(g_{1a}) = 2.7, \ u_i(g_{1a}) = 1.308, \ i = 1, 2, 3, 4$. The total payoffs and utility of the network are $\Pi(g_1) = 10.8$ and $U(g_1) = 5.233$. The adjusted Gini coefficients for the Pleistocene egalitarian social network are $G_\pi(g_{1a}) = 0$ and $G_u(g_{1a}) = 0$.

But hypothetically, let us assume that one of the 4 nodes in the network, for instance node 4, now has the bargaining power. We conduct this thought experiment to see how much it would be worth to a node if she could get bargaining power on a Pleistocene complete economic network.

To do this we need to determine the feasible deviation network set D, its associated payoffs π^D and utilityu^D.

In a complete economic network, every node is connected to everyone else, i.e., each node is within $k = 1$ step away from each other. Therefore the deviation network can assume all possible deviation structures when $k = 1$: node 1, node 2, and node 3 could either form a 3-node clique, indicated by the red-colored $\triangle 123$ in figure 1b, or a line structure such as $2 - 1 - 3$ shown in figure 1c and 1d. We consider both the *first order* and the *second order* deviation scenarios. In first oder deviation scenarios, the essential intermediary demands transfer from the peripheral nodes, which form a commune and redistribute the payoffs from the 1^{st} order deviation network equally. In the second order deviation scenarios, any essential intermediaries among the nodes that have deviated would initiate a second round of bargaining within the 1^{st} order deviation network. Therefore, the 1^{st} *order feasible deviation network set* of the Pleistocene network structure consist of the following 3 cases, in which we give bargaining power to one of the 4 nodes, making her the essential intermediary who can demand transfers from the other 3 nodes:

In case 1, (illustrated by figure 1b), the three nodes form a clique coalition when demanded transfers by the essential intermediary. Second order deviations are not possible in a clique unless we were to arbitrarily assign bargaining power to one of the nodes.

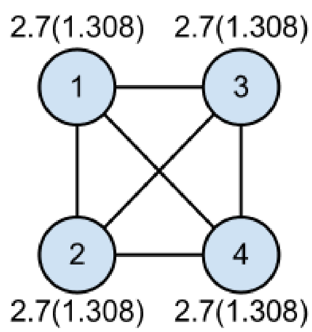
In case 2 (illustrated by figure 1c), the three peripheral nodes form a line where the second order deviation can take place.

In case 3 (illustrated by figure 1d), the three peripheral nodes form a commune of the line structure, where they get primary payoffs from the line structure and subsequently redistribute the payoffs so that each of them get the same payoffs and utility. In other world, we only consider the first order deviation though it is possible for the three peripheral nodes to pursue the second order deviation.

In case 1, each of the three nodes forming the clique would be directly connected the other two and thus receive $\pi_i(g_{1b}) = 2 - 2c = 1.8,\ i = 1, 2, 3$ and $u_i(g_{1b}) = \ln(\pi_i(g_{1b}) + 1) = 1.021,\ i = 1, 2, 3$ in the deviation network. Consequently, it is feasible for node 4 to demand a transfer from each of the 3 nodes that leave them with payoffs arbitrarily larger than $\pi_i(g_{1b})$ and maintain the complete economic network structure 1a. Mathematically, the demanded transfer $T(1b)$ should fulfill the following condition.

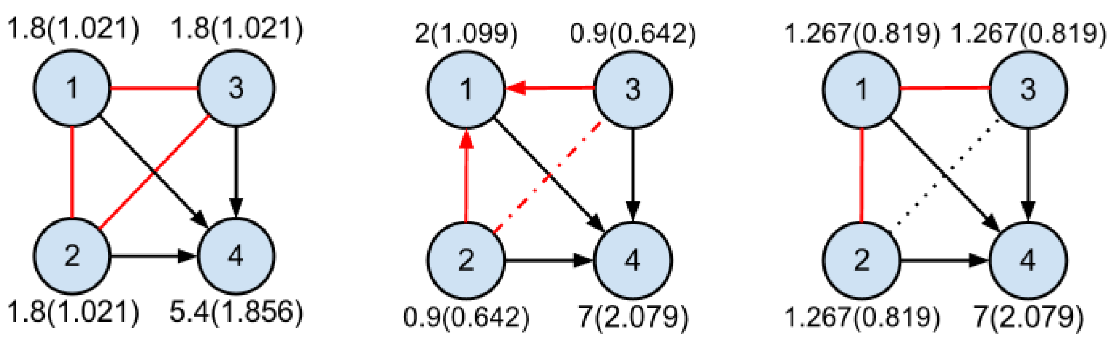
$$\pi_i(g_{1a}) - T(g_{1b}) \geq \pi_i(g_{1b}) \tag{6}$$

Pleistocene complete economic network



(a) Pleistocene complete economic network; $G_\pi = 0$; $G_u = 0$

Feasible deviation networks ($k = 1$; case $1, 2$ and 3)



(b) Bargaining on a Pleistocene network with clique deviation network for the peripheries; $G_\pi = 0.3333$; $G_u = 0.1755$

(c) Bargaining on a Pleistocene network with line and second order deviation network for the peripheries; based on individual payoff and utility, $G_\pi = 0.5988$; $G_u = 0.3562$

(d) Bargaining on a Pleistocene network with a line-structured commune deviation network for the peripheries; $G_\pi = 0.5309$; $G_u = 0.4395$

Figure 1: Maximal inequality in payoffs and utility (in parenthesis) that can be sustained on a 4-node Pleistocene complete economic network. The solid red edges denote the first order deviation structures; the dotted red edges denote the second order deviation structures; the deviation structures titled in red is the most efficient deviation network structure for given k. $c = 0.1$, $\delta_P = 0.1$.

Therefore, the maximal individual transfer to the essential intermediary node 4 is $T_{max}(g_{1b}) = \pi_i(g_{1a}) - \pi_i(g_{1b}) = 0.9$. The payoff and utility of node 4 in case 1 of the late Pleistocene deviation networks are $\pi_4(g_{1b}) = \pi_i(g_{1a}) + 3T_{max}(g_{1b}) = 5.4$, and $u_4(g_{1b}) = \ln(\pi_4(g_{1b}) + 1) = 1.856$ respectively.

The Gini coefficients for case 1 are $G_\pi(g_{1b}) = 0.3333$ for payoffs and $G_u(g_{1b}) = 0.1755$ for utility.

In case 2 and case 3, the peripheral nodes coalesce and form line structure coalition deviations.

To study the distribution and level of inequality of the payoffs and utility of the nodes in each case, we first calculate the primary payoffs and utility for a line structure made up of 3 nodes, which are $\pi_1(g_{1d}) = 2 - 2c = 1.8,\ i = 2, 3$ and $\pi_i(g_{1d}) = 1 + \delta_P - c = 1,\ i = 2, 3$.

In case 2, the central node of the line structure may demand a transfer from the other 2 nodes, as shown in 1c. The deviation network of the other two nodes would be a line on which each will receive payoff equal to $1 - c = 0.9$, the central node would be able to demand $1 - 0.9 = 0.1$ transfer from each of them and receive payoff $2 - 2c + 0.1 \times 2 = 2$ from the structure.

The expected payoff and utility for the 3 peripheral nodes on the Pleistocene complete economic network would be $E(\pi_i(g_{1c})) = \frac{1}{3} \times 2 + \frac{2}{3} \times 0.9 = 1.267$ and $E(u_i(g_{1c})) = \frac{1}{3} \times \ln 3 + \frac{2}{3} \times \ln 1.9 = 0.794,\ i = 1, 2, 3$; the Gini coefficients of the network 1c based on individual payoffs and utilities are $G_\pi(g_{1c}) = 0.5988$ and $G_u(g_{1c}) = 0.3562$.

In case 3, the peripheral nodes form a commune in the shape of a line. They agree to redistribute among themselves equally the payoffs that arise from such a line structure. Each periphery receives the expected payoff when they have an equal probability $p = \frac{1}{3}$ to be the central node of the first order deviation line. As in case 1, node 4 can demand a transfer that leaves the peripheries with payoffs and utilities arbitrarily better than what they would get from the deviation network: $\pi_i(g_{1d}) = E(\pi_i(g_{1d})) = \frac{1}{3} \times 1.8 + \frac{2}{3} \times 1 = 1.267$ and $u_i(g_{1d}) = E(u_i(g_{1d})) = \ln(E(\pi_i(g_{1d})) + 1) = 0.819,\ i = 1, 2, 3$.

Meanwhile, node 4 is capable to demand from each of the peripheries $T_{max}(g_{1d}) = \pi_i(g_{1a}) - \pi_i(g_{1d}) = 1.433$, which bring the payoffs and utility of node 4 up to $\pi_4(g_{1d}) = \pi_i(g_{1a}) + 3T_{max}(g_{1d})) = 7$ and $u_4(g_{1d}) = \ln(\pi_4(g_{1d}) + 1) = 2.080$.

The Gini coefficients of the network are $G_\pi(g_{1d}) = 0.5309$ and $G_u(g_{1d}) = 0.2781$.

Notice that case 1 not only is the most efficient in the deviation network set, but also Pareto dominates both case 2 and case 3 in expected payoffs and utility. This does not always hold true. If δ_P and c take the value such that the end nodes of the deviation line structure receive higher primary payoffs than the central node, that is, when $c + \delta_P > 1$, then case 3 instead of case 1 would be the Pareto dominant deviation network structure. In addition, when $c + \delta_P = 1$, despite the difference in the political structure, the distribution of payoffs and utility of all the 3 cases would be the same.

The analysis of the 4-node late Pleistocene network might give the impression that the introduction of political hierarchy to an originally egalitarian complete economic network would result in significant levels of inequality on the network. Here we see if the same inference can be made for a larger network. We consider a network of 20 nodes, which is similar to the size of a typical Pleistocene community.

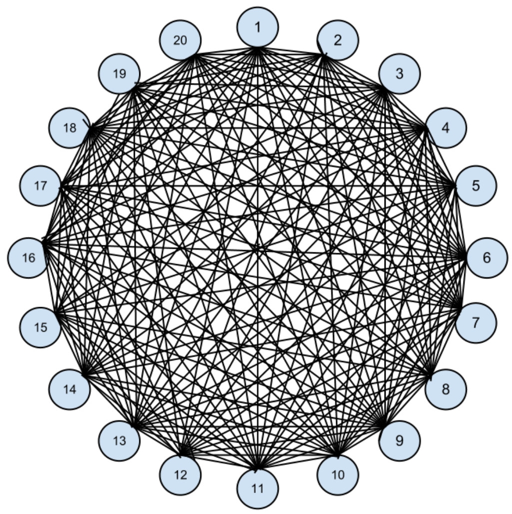


Figure 2: A 20-node Pleistocene complete economic network

Figure 2 illustrates a late Pleistocene complete economic network. Without changing the social condition assumption that $c = 0.1, \delta_P = 0.1$, we have $\pi_i(g_2) = 19 - 19c = 17.1,\ i = \{1, 2, ..., 20\}$ and $u_i(g_2) = \ln(\pi_i(g_2) + 1) = 2.896,\ i = \{1, 2, ..., 20\}$. The total payoffs and utility of the network are $\Pi(g_2) = 17.1 \times 20 = 342$ and $U(g_2) = 2.896 \times 20 = 57.9$, and the adjusted Gini coefficients for the egalitarian social network above are $G_\pi(g_2) = 0;\ G_u(g_2) = 0$.

Suppose one of the twenty nodes on the above network structure demands transfer from the other19 nodes, and suppose the 19 nodes coalesce into a 19-node clique structure. Each node in the deviation network are connected to the other 18 nodes in the clique and receives $\pi_i(g_{deviation}) = 18 - 18c = 16.2,\ i = \{2, 3, ..., 20\}$ and $u_i(g_{deviation}) = \ln(\pi_i(g_{deviation}) + 1) = 2.845,\ i = \{2, 3, ..., 20\}$

Consequently, it is feasible for node 1 to demand a transfer from each of the 19 nodes that leave them with payoffs arbitrarily larger than $\pi_i(g_{deviation})$ and maintain the complete economic network structure 2. The maximal individual transfer value from the peripheries to the essential intermediary is $T_{max}(g_{deviation}) = \pi_i(g_2) - \pi_i(g_{deviation}) = 17.1 - 16.2 = 0.9$.

The payoff and utility for the essential intermediary are $\pi_1(g_{deviation}) = \pi_i(g_{deviation}) + 19 \times T_{max}(g_{deviation}) = 17.1 + 19 \times 0.9 = 34.2$ and $u_1(g_{deviation}) = \ln(\pi_1(g_{deviation}) + 1) = 3.561$. The total payoff and utility for the deviation network are $\Pi(g_{deviation}) = 16.2 \times 19 + 34.2 = 307.8$ and $U(g_{deviation}) = 2.845 \times 19 + 3.561 = 57.614$. The adjusted Gini coefficients for structures illustrated by figure deviation are $G_\pi(g_{deviation}) = 0.0526$ and $G_u(g_{deviation}) = 0.0124$.

The small Gini coefficients indicate that on a larger Pleistocene complete economic network, a node would not be able to gain much through bargaining with others. Since in real world, the assumption of the essential intermediary position incurs cost, therefore it is unlikely for economic inequality and political hierarchy to arise in the Peistocene complete economic network. Specifically for the 20-node network, the cost to assume the bargaining power for node 1 must be less than the total transfer she could demand from the peripheries, i.e., $19 \times 0.9 = 17.1$, for there to be an interest for her to assume the power.

2.2 Holocene networks

The Pleistocene epoch was followed by the Holocene epoch. Many people become farmers and live in geographically dispersed farm houses, making the cost of social linkage maintenance higher. The substance transmitted on the social networks were mostly storable goods such as grains and livestock as well as rights of visitation in times of need. As a result, indirect economic connections took on higher values. We assume $\delta_H = 0.8$ and $c = 0.3$ in our analysis. Bowles (2009) hypothesized that the Holocene economic network structures would be less dense than the Pleistocene economic networks and that the prototypical early Holocene network structure of 4 nodes is a star. Suppose node 1 is central. Figure 3a illustrates the network structure which gives primary payoff $\pi_1(g_{3a}) = 3 - 3c = 2.1$ and utility $u_1(g_{3a}) = \ln(\pi_1(g_{3a}) + 1) = 1.131$.

Each of the remaining nodes maintains 1 direct link with node 1 and benefits indirectly from the other two nodes. This give them payoff $\pi_i(g_{3a}) = 1 + 2\delta_H - c = 2.3,\ i = 2, 3, 4$ and utility $u_i(g_{3a}) = \ln(\pi_i(g_{3a}) + 1) = 1.194,\ i = 2, 3, 4$. The total primary payoffs and utility for a 4-node star economic network are $\Pi(g_3) = 9$ and $U(g_3) = 4.713$ respectively.

The adjusted Gini coefficients are $G_\pi(g_{3a}) = 0.0222$ and $G_u(g_{3a}) = 0.0133$.

In an early Holocene star economic network, the central node assumes the highest betweenness and therefore poses as the essential intermediary in the political structure. When $k = 1$, each of the peripheral singly deviates. When $k = 2$, there are two possible deviation structures. node 1, node 2, and node 3 can either form a 3-node clique, indicated by the red-colored $\triangle 234$ in figure 3b, or a line structure such as $2 - 3 - 4$ shown in figure 3e and 3d. We discuss both the first order and the second order deviation scenarios for line deviation structure. These 3 cases under $k = 2$ form the 2^{nd} *order deviation network set* of the Holocene network structure. In total, we have 4 deviation network possibilities:

In case 1 (illustrated by figure 3b), $k = 1$, the three peripheral nodes are isolated if they chose to deviate.

In case 2 (illustrated by figure 3c), the three nodes form a clique deviation network. Second order deviations are not possible in a clique.

In case 3 (illustrated by figure 3d), the three peripheral nodes form a line where the second order deviation can take place.

In case 4 (illustrated by figure 3e), the three peripheral nodes form a commune line structure, where they get primary payoffs from the line structure and subsequently redistribute the payoffs so that each of them get the same payoffs and utility.

In case 1, the three peripheral nodes' deviation payoffs and utility are both 0, which enables node 1 to demand transfers from each of the peripheries up to the level that they each receives some payoffs arbitrarily bigger than 0. Each of the peripheral nodes get payoff $\pi_i(g_{3b}) = 0$ and utility $u_i(g_{3b}) = ln(\pi_i(g_{3b}) + 1) = 0$. The maximal transfer node 1 can demand from each of the peripheral nodes is $T_{max}(g_{3b}) = \pi_i(g_{3a}) - \pi_i(g_{3b}) = 2.3,\ i = 2, 3, 4$, which gives node 1 the payoff $\pi_1(g_{3b}) = \pi_1(g_{3a}) + 3 \times T_{max}(g_{3b}) = 2.1 + 3 \times 2.3 = 9$ and utility $u_1(g_{3b}) = \ln(\pi_1(g_{3b}) + 1) = 2.303$.

The adjusted Gini coefficients for the case 1 deviation network are $G_\pi(g_{3b}) = 1$ and $G_u(g_{3b}) = 1$.

In case 2, each of the three nodes forming the clique would be directly connected with the other two and thus receive deviation payoff $\pi_i(g_{3c}) = 2 - 2c = 1.4,\ i = 2, 3, 4$ and utility $u_i(g_{3c}) = \ln(\pi_i(g_{3c}) + 1) = 0.875,\ i = 2, 3, 4$.

Consequently, it is feasible for node 1 to demand a transfer from each of the 3 nodes that leave them with payoffs arbitrarily larger than $\pi_i(g_{3c}) = 1.4$ and maintain the star economic network structure 3a. The maximal transfer value from each individual peripheral nodes to the essential intermediary is $T_{max}(g_{3c}) = \pi_i(g_{3a}) - \pi_i(g_3) = 2.3 - 1.4 = 0.9$. Each of the peripheral nodes receives payoff $\pi_1(g_{3c}) = \pi_i(g_{3a}) + 3 \times T_{max}(g_{3c}) = 2.1 + 3 \times 0.9 = 4.8$ and utility $u_1(g_{3c}) = \ln(\pi_1(g_{3c}) + 1) = 1.758$.

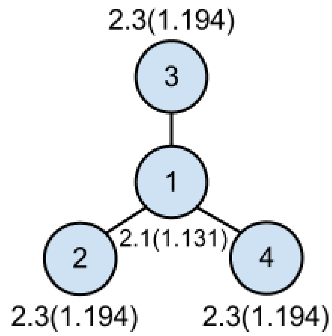
The Gini coefficients for the second case of deviation network of the Holocene star economic network are $G_\pi(g_{3c}) = 0.3778$ and $G_u(g_{3c}) = 0.2015$.

In case 3 and case 4, the peripheral nodes coalesce and form line coalition deviations.

The primary payoffs for a line structure made up of 3 nodes are $\pi_3(g_{3e}) = 2 - 2c = 1.4$ for the central node and $\pi_i(g_{3e}) = 1 + \delta_H - c = 1.5,\ i = 2, 4$ for the two peripheral nodes.

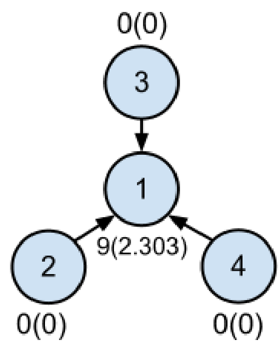
In case 3, the central node of the line structure initiates the second order deviation negotiation and demands transfer from the other 2 nodes, as shown in 3d. Because the

Holocene star economic network



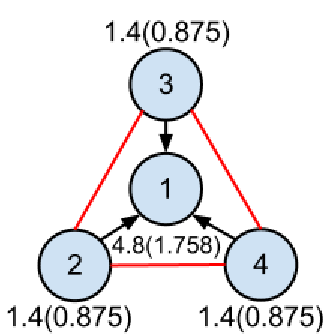
(a) Holocene star economic network; $G_\pi = 0.0222$; $G_u = 0.0133$

Feasible deviation networks ($k = 1$; case 1)

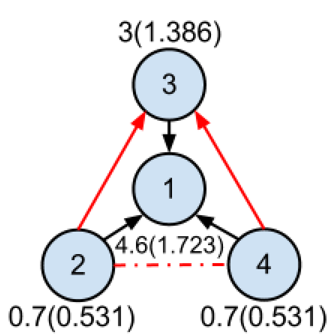


(b) First order deviation bargaining on a Holocene network; $G_\pi = 1$; $G_u = 1$

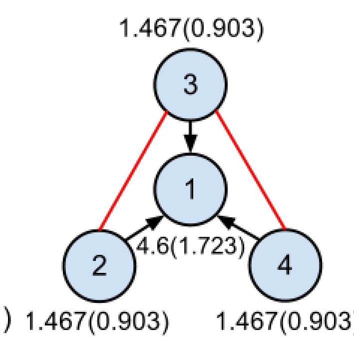
Feasible deviation networks ($k = 2$; case $2, 3$ and 4)



(c) Bargaining on Holocene network with clique deviation network for the peripheries; $G_\pi = 0.3778$; $G_u = 0.2015$



(d) Bargaining on a Holocene network where the peripheries form a coalition line and conduct second order deviation bargaining; $G_\pi = 0.5185$ $G_u = 0.3541$



(e) Bargaining on a Holocene network where the peripheries form a commune in the line strutter; $G_\pi = 0.3481$; $G_u = 0.1850$

Figure 3: Maximal inequality in payoffs and utility (in parenthesis) that can be sustained on a 4-node Holocene complete economic network. The solid red edges denote the first order deviation structures; the dotted red edges denote the second order deviation structures; the deviation structures titled in red is the most efficient deviation network structure for given

deviation network of the other two nodes would be a line on which each will receive payoff equal to $1 - c = 0.7$, the central node of the line would be able to demand $1.5 - 0.7 = 0.8$ transfer from each of them and receive payoff $1.4 + 2 \times 0.8 = 3$ from the structure.

The expected payoff and utility for the 3 peripheral nodes on the economic network are $E(\pi_i(g_{3d})) = \frac{1}{3} \times 3 + \frac{2}{3} \times 0.7 = 1.467$ and $E(u_i(g_{3d})) = \frac{1}{3} \times \ln 0.531 + \frac{2}{3} \times \ln 1.386 = 0.816, \; i = 2, 3, 4$.

The Gini coefficients of the network 3d based on individual payoffs and utilities are $G_\pi(g_{3d}) = 0.5185$ and $G_u(g_{3d}) = 0.3541$.

In case 4, the peripheral nodes form a commune in the shape of a line. They redistribute among themselves the payoffs equally. Each of the 3 peripheries receives the expected payoff $\pi_i(g_{3e}) = E(\pi_i(g_{3e})) = \frac{1}{3} \times 1.4 + \frac{2}{3} \times 1.5 = 1.467, \; i = 1, 2, 3$ and the associated utility $u_i(g_{3e}) = \ln(\pi_i(g_{3e}) + 1) = 0.903, \; i = 1, 2, 3$.

Node 1 can demand from each of the 3 peripheral nodes $T_{max}(g_{1d}) = \pi_i(g_{3a}) - \pi_i(g_{3e}) = 2.3 - 1.467 = 0.833$ and gets $\pi_1(g_{3e}) = \pi_i(g_{3a}) + 3 \times T_{max}(g_{3e})) = 4.6$ and $u_1(g_{3e}) = \ln(\pi_1(g_{3e}) + 1) = 1.723$. The Gini coefficients of the network are$G_\pi(g_{3e}) = 0.3481$ and $G_u(g_{3e}) = 0.1850$.

Notice that case 4 is the most efficient in the 2-step deviation network set and Pareto dominates case 2 and case 3 in expected payoffs and utility. However, if δ_H and c take the value such that the end nodes of the line structures receives lower primary payoffs than the central nodes of the line, i.e., $c + \delta_H < 1$, then case 2 instead of case 3 would be the dominant deviation network structure. In addition, when $c + \delta_H = 1$ despite the difference in the political structure, the distribution of payoffs and utility of all the 3 cases when $k = 2$ would be the same.

Whether in Pleistocene or Holocene networks, the essential intermediary in the first order deviation process would only be able to get the transfer value equal the difference between the remaining nodes' original structure utility payoff and the utility pay-off that maximizes their deviation payoff collectively.

Table 1 list the summary of the 4-node static analysis. Notice the political structure plays an important role in the inequality outcome. The political power bestowed on one of the node on the Pleistocene network increases the inequality on the network greatly. On early Holocene networks, $k = 1$ produces vast inequality on the network while $k = 2$ result in much more equal payoff and utility distributions.

Network charisteristics	Deviation nodes π, u	Maximum expected transfer	Essential Intermediary π, u	Difference in π, u b/t EI and peripheries	Gini coefficients G_π, G_u
Pleistocene (symmetric)	n/a	0	0	0	0,0
Pleistocene (complete $\rightarrow$ clique)	1.8, 1.1021	0.9	5.4, 1.856	3.6, 0.835	0.3333, 0.1755
Pleistocene (complete $\rightarrow$ line) first order deviation only	1.267, 0.819	1.433	7, 2.079	5.733, 1.260	0.5309, 0.4395
Pleistocene (complete $\rightarrow$ line) second order deviation	1.267, 0.794	1.433	7, 2.079	5.733, 1.285	0.5988, 0.3562
Holocene (k=1, star $\rightarrow$ autarchy)	0, 0	2.3	9, 2.303	9, 2.303	1, 1
Holocene (k=2, star $\rightarrow$ clique	1.4, 0.875	0.9	4.8, 1.758	3.4, 0.833	0.3778, 0.2015
Holocene (k=2, star $\rightarrow$ line) first order deviation only	1.467, 0.903	0.833	4.6, 1.723	3.133, 0.820	0.5309, 0.4395
Holocene (k=2, star $\rightarrow$ line) second order deviation	1.467, 0.816	0.833	4.6, 1.723	3.133, 0.907	0.5185, 0.3541

Table 1: **Political and economic inequality on Pleistocene and Holocene networks.** Source: text. When the essential intermediary (all except the first row) demands the maximum transfer (expected values in column 3) from the peripheries and receives the payoff and utility in column 4, the peripheries get expected payoffs and utility in column 2, which is the most they could make in the associated deviation network. We assume $c_P = 0.1, \delta_P = 0.1$ for the Pleistocene networks and $c_H = 0.3, \delta_H = 0.8$ for the Holocene networks. The last column gives the adjusted gini coefficients for payoffs and utility in each of the network structures.

2.2.1 Inequality on a larger Holocene network

We consider a Holocene star economic network of 20 nodes. The primary payoffs and utility for the central node 1 is $\pi_1(g) = 19 - 19c = 13.3$. Each of the 19 peripheries receives payoff $\pi_i(g) = 1 + 18\delta_H - c = 15.1,\ i = \{2, ..., 20\}$.

The total payoffs and utility for the 20-node early Holocene star economic network are $\Pi(g) = 13.3 + 19 \times 15.1 = 300.2$ and $U(g) = 2.660 + 19 \times 2.779 = 55.461$, and the adjusted Gini coefficients for the primary payoffs of the network structure are $G_\pi(g) = 0.0060$ and $G_u(g) = 0.0021$.

Node 1 is the essential intermediary of the star economic network. It is feasible for node 1 to demand transfers from her peripheries. As we have shown in the previous section, under the Holocene social conditions $\delta_H = 0.8$ and $c = 0.3$, the line commune, or a commune sub-star, is the dominant network structure when $k = 2$, so we study the case where the remaining 19 nodes coalesce into a 19-node star structure and redistribute their payoffs equally.

First we calculate the primary payoffs for the central node (let us assume it to be node 2) and the peripheries of the star network structure. The primary payoff for the central node is $\pi_2(g_{deviation}) = 18 - 18c = 12.6$, and the primary payoffs for the peripheral nodes are $\pi_i(g_{deviation}) = 1 + 17\delta_H - c = 14.3,\ i = \{3, ..., 20\}$. The expected payoff of the each node on the deviation network is $\pi_i(g_{deviation}) = \frac{12.6+18\times14.3}{19} = 14.211,\ i = \{2, 3, ..., 20\}$. Each of them receives $u_i(g_{deviation}) = \ln(\pi_i(g_{deviation}) + 1) = \ln(14.211 + 1) = 2.722,\ i = \{2, 3, ..., 20\}$ in utility.

Consequently, it is feasible for node 1 to demand a transfer from each of the 19 nodes that leave them with payoffs arbitrarily larger than $\pi_i(g_{deviation})$. The maximal individual transfer value to the essential intermediary node 1 is $T_{max}(g_{deviation}) = \pi_i(g) - \pi_i(g_{deviation}) = 15.1 - 14.211 = 0.889$. The payoff and utility for the essential intermediary are $\pi_1(g_{deviation}) = \pi_i(g_{deviation}) + 19 \times T_{max}(g_{deviation}) = 13.3 + 19 \times 0.889 = 30.191$ and $u_1(g_{deviation}) = \ln(\pi_1(g_{deviation}) + 1) = 3.440$.

The adjusted Gini coefficients for structures are $G_\pi(g_{deviation}) = 0.0532$ and $G_u(g_{deviation}) = 0.0130$.

Similar to the result of the late Pleistocene larger network, the small Gini coefficients indicate that on a larger Holocene star economic network, any individual node would not be able to gain much through bargaining with others. For the 20-node star network, the cost to assume the bargaining power for node 1 must be less than the total transfer she could demand from the peripheries, i.e., $19 \times 0.899 = 17.081$, for there to be an interest for her to assume the power.

Notice that the $G_\pi(g_{deviation})$ and $G_u(g_{deviation})$ values for the 20-node networks under both the Pleistocene and the Holocene contexts are extremely close. It might be the case that such values are common among dominant deviation network structures of similar sizes.

Finally, if $k = 1$, the deviation payoff for each of the 19 peripheral nodes would receive 0 and the essential intermediary will get the entire outcome of the network. This will result in extremely unequal payoff and utility outcomes. Again, we see the variation of k has tremendously impact on the maximum degree of inequality on a network.

2.3 Effects of structure and social assumptions on inequality of networks

The analysis of the political coalitions on the Pleistocene and the Holocene networks give us ideas about how political structure could affect economic structure given the prototypical economic structures. What are the net effects of network structure on the inequality distribution of a network? To single out these effects, I extend the analytical process in the previous sections to the hypothetical Pleistocene star network and the hypothetical Holocene complete network. Table 2 shows the results.

Both the Pleistocene complete network and the Holocene hypothetical complete network have equal payoffs and utility distribution. Similarly, both star primary payoffs, be it realistic or hypothetical, show moderate degree of inequality. Once bargaining power is taken into account and $k = 1$, deviation networks of the early Holocene star sustain extreme inequality - $G = 1$. When bargaining takes place and $k = 2$, the hypothetical Pleistocene star network has the same payoff distribution as the primary case, since the essential intermediary node 1 would be better off not demanding transfer from the other 3 nodes, in which case the three would deviate and realize that they, the peripheries, could profit by deviating from the essential intermediary and even demand transfer from the politically powerful node. However, the three cannot demand transfers back from the essential intermediary according to our rules concerning bargaining. Meanwhile, knowing the consequence, the essential intermediary 1 would not initiate the bargaining process, which makes happen the primary payoff distribution for the $k = 2$ deviation cases.

In the final analysis, network structure matters. Complete network give egalitarian distribution while star networks sustain higher levels of inequality. The sustainable inequality on a network is the highest when the structure is a star and $k = 1$.

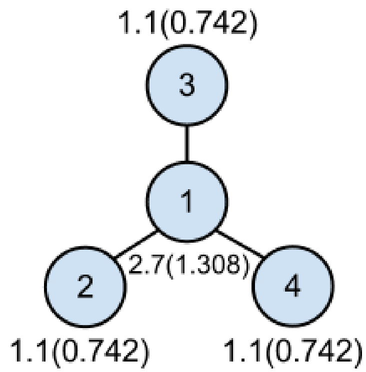
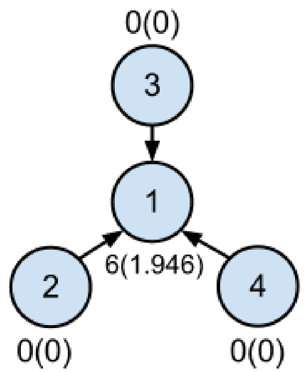
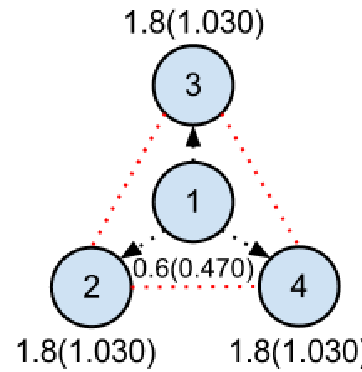
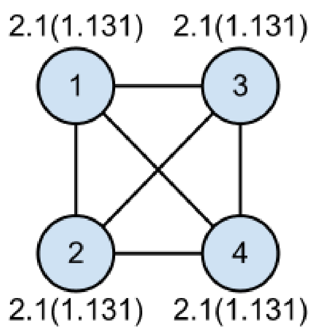
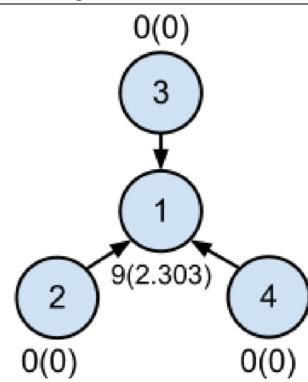
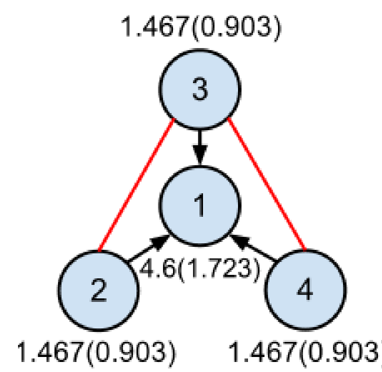
	complete economic network	star economic network	
	primary	$k = 1$	$k = 2$
$c^P = 0.1$ $\delta^p = 0.1$	 $\Pi = 10.8;\ U = 5.2333$ $G_{pi} = 0; G_u = 0$	 $\Pi = 6;\ U = 3.534$ $G_{pi} = 1; G_u = 1$	 Deviation is not feasible. Node 1 does not demand transfers.
$c^H = 0.3$ $\delta^H = 0.8$	 $\Pi = 8.4;\ U = 4.526$ $G_{pi} = 0; G_u = 0$	 $\Pi = 9;\ U = 2.303$ $G_{pi} = 1; G_u = 1$	 $\Pi = 9;\ U = 4.432$ $G_{pi} = 0.3481; G_u = 0.1850$

Table 2: Payoff and utility comparison of complete and star network under Pleistocene and Holocene social condition assumptions. Figures with red-color summary characteristics are realistic scenarios discussed in previous sections and are listed here for comparison with the hypothetical cases. For $k = 2$ in the star network, we only list the Pareto superior case in the deviation network set. Line in red denotes political coalition; dash line denotes political coalition that would not actually take place.

3 Evolution of economic inequality and political hierarchy on 4-node social networks

3.1 The model

We consider two dynamics in the evolution of economic inequality and political hierarchy on social networks made up of 10 network communities of size 4.

The first dynamic is *within group transition*: every time step, the model randomly picks 2 nodes in each of the 10 communities, and determine if they will benefit from a change in the status of their adjacency. The formation of links in the social network follows *pairwise stability* (see Jackson, 2008), where links can only be formed between two nodes through mutual consent, but either node adjacent to an existent edge can sever the link between them. Agents make choices of network formation based on expected utility from either primary payoff or bargaining payoff. We introduce a new utility function that allows for control of the function concavity.

$$u_i(g) = 1 - e^{-a\pi_i}, i \in \mathbb{N} \tag{7}$$

with π_i being node i's payoff. Notice that

$$a = -\frac{u''}{u'} \tag{8}$$

corresponds to the concavity of the utility function. As we can see in figure 4, larger a value corresponds to greater concavity in utility function, and greater utility function results in higher total utility for more equalized society among the ones with the same total payoff.

The second dynamic is *between group competition*. I use μ to denote the frequency of competitions. Then every $\frac{1}{\mu}$ time steps, the model randomly selects two communities to enter between group competitions. The community with the higher total utility, or fitness, wins the competition, and replace the social network structure of the losing community with its own.

Two types of people interact in the model: λ fraction of the nodes are far-sighted agents and the remaining $(1 - \lambda)$ fraction of the agents are short-sighted. The far-sighted agents anticipate the effect of their actions on bargaining power and the network. They foresee their post bargaining payoffs according to the bargaining process in section 2. Meanwhile, the short-sighted agents consider only the primary payoffs from the network structure. Correspondingly, there are also two types of world - the complicated world, where people are awarded with bargaining payoff, and the simple world, where people are awarded with pri-

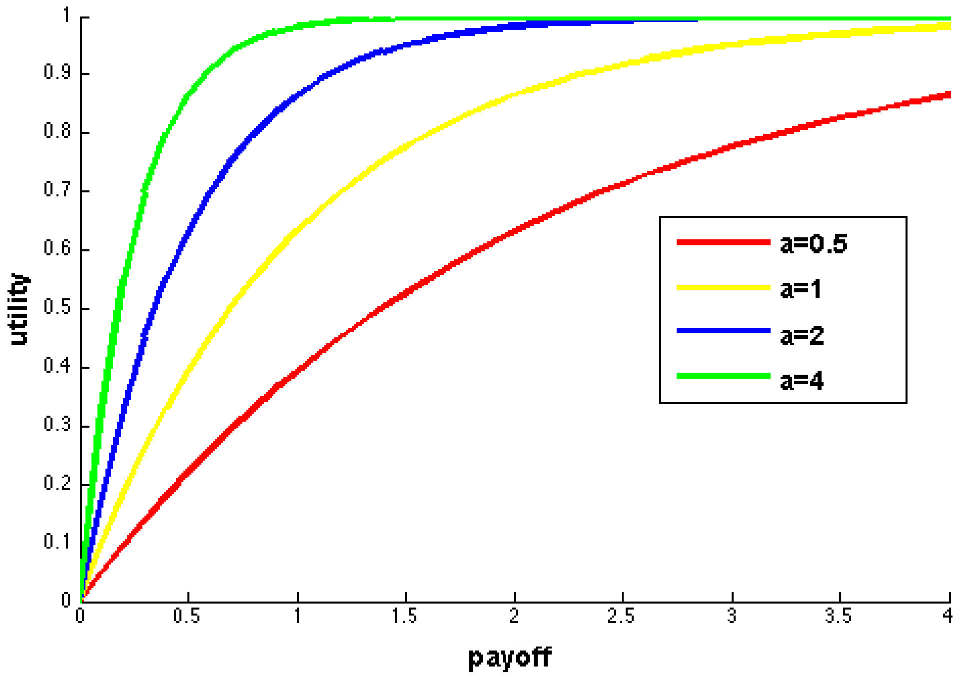


Figure 4: Concavity of the utility function $u_i(g) = 1 - e^{-a\pi_i}, i \in \mathbb{N}$ with $a = 0.5, 1, 2, 4$

mary payoff in accord to the network structure.

Finally, we introduce the concept of network accessibility.

Definition 1 *Network g_i is said to be accessible from g_j if g_i can be attained from g_j through transitions in accord to pairwise stability rules.*

For example, suppose $c < 1$, structure 3 is accessible from structure 2 in the map of all possible network structure types of 4 nodes numbered in figure 5. Suppose the two individual nodes in structure 2 are picked. Since $c < 1$, both nodes would benefit $1 - c > 0$ from forming a link between them. According to the pairwise stability rules, the link would be formed and the structure type of the network would transition from 2 to 3.

However, in order for this to happen, exactly the two individual nodes would have to be randomly picked. The probability for the event is $\frac{1}{6}$. We call the probability of a structure transition from structure type i to adjacent structure type j the *arrival probability*, denoted by p_{ji}.

In addition, for each of the structure type, there is a persistence probability $sigma_i$ that structure type i undergoes within group transition process but remain in the same structure type afterward. The expected duration of a certain type of network structure in the transition process is then $\frac{1}{1-\sigma_i}$

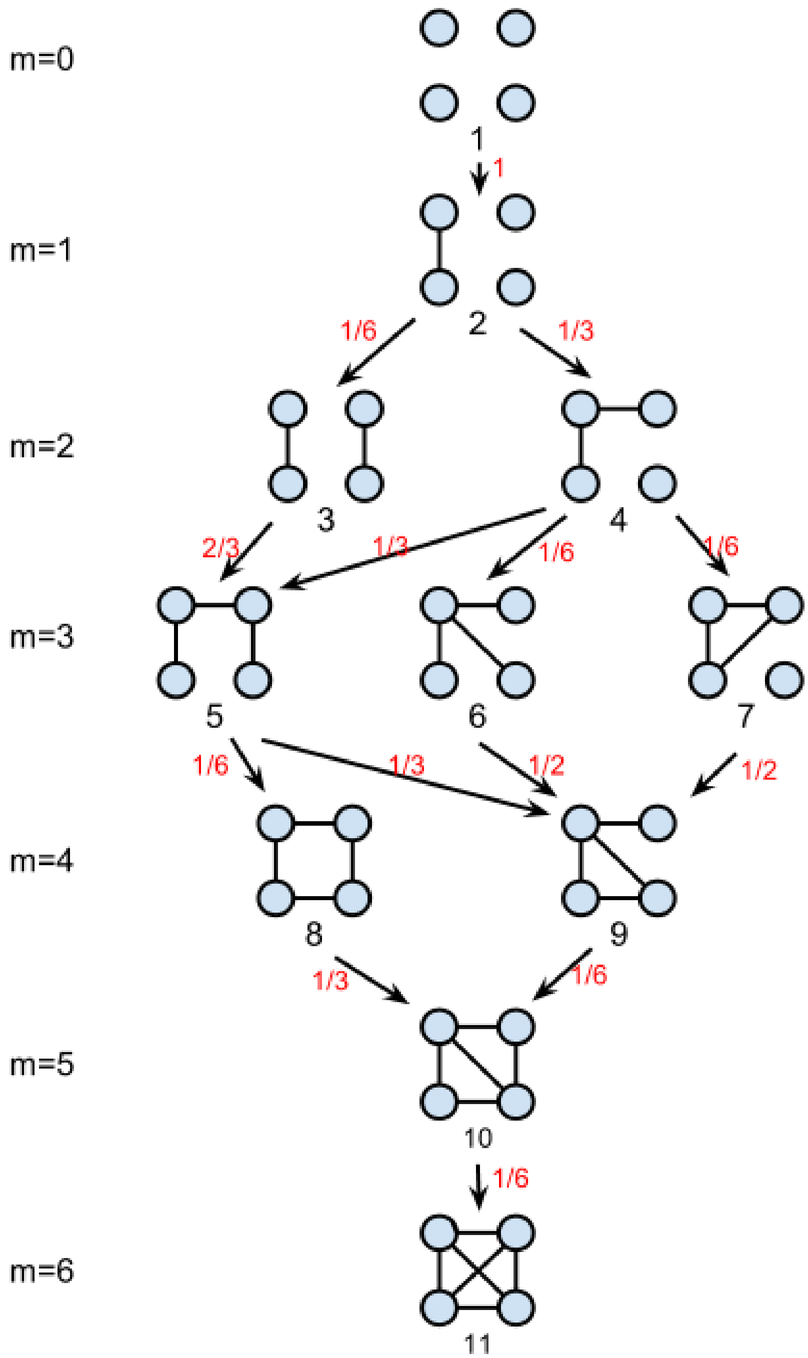


Figure 5: Map of 4-node network structures and accessible transition routes under the conditions $c < 1$ and $c + \delta < 1$, as well as arrival probabilities between neighboring network structures in red.

	1	2	3	4	5	6	7	8	9	10	11	H
1	0											0
2	1	2										2
3	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{3}{2}$									$\frac{1}{4}$
4	$\frac{1}{3}$	$\frac{1}{3}$		2								$\frac{2}{3}$
5	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{3}$	$\frac{1}{3}$	2							2
6	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{6}$	$\frac{1}{6}$		2						$\frac{1}{3}$
7	$\frac{1}{18}$	$\frac{1}{18}$		$\frac{1}{6}$			2					$\frac{1}{3}$
8	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{18}$	$\frac{1}{6}$			3				$\frac{1}{2}$
9	$\frac{7}{54}$	$\frac{7}{54}$	$\frac{2}{9}$	$\frac{5}{18}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$		6			8
10	$\frac{11}{324}$	$\frac{11}{324}$	$\frac{2}{27}$	$\frac{7}{108}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{1}{6}$	6		4
11	$\frac{11}{1944}$	$\frac{11}{1944}$	$\frac{1}{81}$	$\frac{7}{648}$	$\frac{1}{54}$	$\frac{1}{72}$	$\frac{1}{18}$	$\frac{1}{36}$	$\frac{1}{6}$	$\frac{1}{6}$	∞	∞

Table 3: Arrival probability, expected duration (red), and the 1-step accessibility H for each of the structure type. The numbers in blue are multi-step arrival probability calculated from arrival probability. Unfilled cells indicate unfeasible transitions.

Table 3 lists the arrival probability p_{ij} and persistence probability r_i based on primary payoffs. The persistence probabilities are located across the diagonal axis of the table and are in red.

The final column of the table shows the $1-$*step accessibility* of each structure type. The 1-step accessibility H is defined by

$$H_i = \frac{\sum_j p_{ij}}{\frac{1}{1-\sigma}}. \tag{9}$$

3.2 Model inputs

Here is a summary table of the model inputs:

Fixed variables

$N = 4$ – number of nodes

$M = 10$ – number of communities

Varied variables

c – cost of maintaining link

δ – value of indirect connections

λ – fraction of far-sighted people

a - concavity of utility functions

μ – frequency of between group competitions

Whether the agents are rewarded with primary payoff or bargaining payoff

3.3 Simulation results

We run the simulation for controlled set of parameters and vary λ, the fraction far-sighted agents in the population. We find that short-sighted population converges to the efficient network outcomes when they are rewarded with the primary payoffs that they anticipate. Meanwhile, heterogeneous population produces divergent network outcomes. Figure 6 illustrates this result.

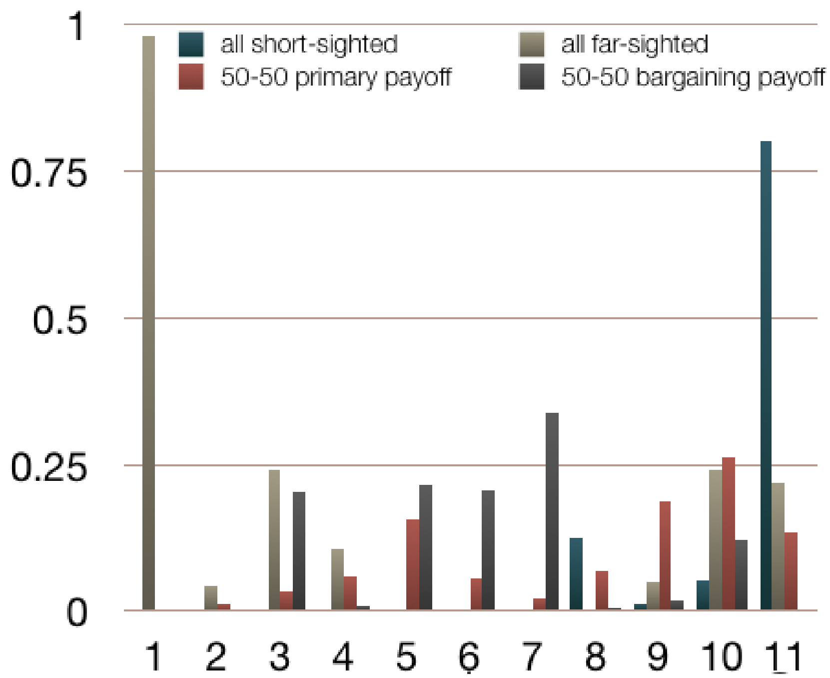


Figure 6: Simulation results when varying λ – fraction of far-sighted people in the population. Structure type code corresponds to the network structure types in figure 5

The 4 simulations that are shown in figure 6 are attained when seeding the model with $c = 0.1, \delta = 0.1, a = 1$, $\mu = 0.5$, and run the model for 200 time-steps. We run the model under 4 different scenarios:

Scenario 1: $\lambda = 0$, all agents are short-sighted, and they receive primary payoffs according to the network structure after each step of evolutions.

Scenario 2: $\lambda = 1$, all agents are far-sighted, and they receive bargaining payoffs as analyzed in section 2.

Scenario 3: $\lambda = 0.5$, 50% of the agents are far-sighted and the other 50% are short-sighted, all agents receive primary payoffs.

Scenario 4: $\lambda = 0.5$, 50% of the agents are far-sighted and the other 50% are short-sighted, all agents receive bargaining payoffs.

As in figure 6, 73.8% of the short-sighted agents stay in the complete network structure over the 200 time steps. Towards the end, all agents stay in the complete network structure. The network structure evolve to the most efficient type through the individual level evolutions. In scenario 2, we see a large number of far-sighted agents enter empty networks. This is because fa-sighted agents anticipate bargaining payoff. As we saw in section 2, most peripheral locations on a network give 0 payoff and hence these agents are indifferent towards any network structure types, which are many, that give them 0 in potential payoffs. Scenario 3 and scenario 4 simulations produces a spectrum of network structure outcomes, which illustrates our observation that heterogeneous population composition give rise to diverse network structure distributions.

Secondly, we run simulations with different frequencies of between group competitions. We find that within network transitions and between network competitions work in opposite directions under certain conditions. Figure 7 illustrates this result.

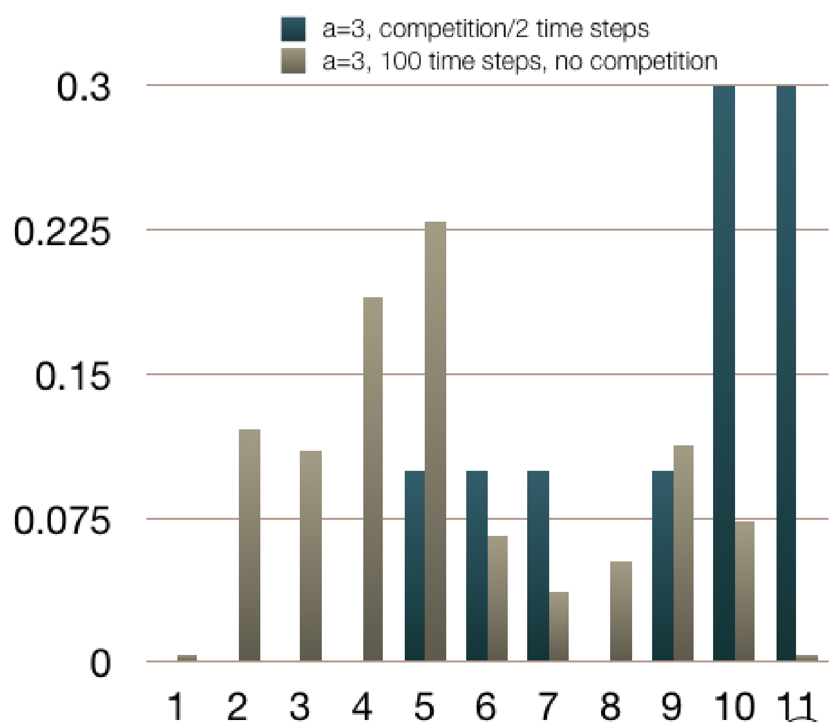


Figure 7: Simulation results when varying μ – frequency of between group competitions. Structure type code corresponds to the network structure types in figure 5

The parameters that produce figure 7 are $c = 0.3, \delta = 0.8, a = 3$, and $\lambda = 0.5$. I vary the frequency of between group competitions over 100 time steps. For the results in figure 7, I run the model with $\mu = 0.5$, and $\mu = 0.01$. With $\mu = 0.5$, between group competitions happen every 2 time steps. The frequent conflicts give high network structure concentrations in network type 10, the network structure with 5 edges, and 11, the complete network. The results for $\mu = 0.01$ during with no between group competition happened are different. We see over 40% of the network structure outcomes lie in type 4 and 5, which are star-like networks.

Notice under the social conditions $c = 0.3$ and $\delta = 0.8$, star is the most efficient network structure as we discussed in section 2. But complete networks are more equal and higher in total utility and fitness. In the event of between group competitions, the complete networks will beat the more efficient star networks and thus increase its representation in the network type outcome distributions.

Although complete network or star network is the most efficient network structure for most combinations of c and δ, the majority of the network outcomes are nonetheless neither complete nor star most of the time. Instead, the concentration of network structure outcomes fall in star-like structures 4 and 5, and almost complete structures 9 and 10 most of the time.

We notice that all of the above nodes are high in centrality on the network transition map and have high accessibility in table 3. The correlation between network type distribution and network types accessibility leads to the conclusion that the evolution of economic and political hierarchy not only depends on economic and political structures, but the accessibility of network structure among the possible network structure set as well!

4 Conclusion

In section 2, I conducted static analysis and studied the maximal level of inequality that is sustainable on 4-node social networks. We find that both the economic structure and the political structure that it induces, independently have an immense impact on the distribution outcomes of payoff and utility and hence degree of inequality on network structures. Star network with $k = 1$ bargaining gives the most unequal payoff distribution. When bargaining is taken into consideration and $k = 2$, we find that larger social networks sustain less degree of inequality.

In section 3, I presented the model of evolution of economic inequality and political hierarchy on 4-node communities. The model results show that community networks converge to the efficient outcomes when the agents on the network are rewarded with anticipated payoffs. Heterogeneous population produces an array of various network structure outcomes.

We find that within network transitions and between network competitions work in opposite directions under certain conditions. Finally, the evolution of economic and political hierarchy not only depends on economic and political structures, but the accessibility of network structure among the possible network structure set as well.

We thus conclude the research for the summer, and hope that the reader is encouraged to explore the subject further. Future research directions include analysis and simulation of networks of larger sizes and introducing stochastic elements to the process of forming and breaking social links on the network. It would be interesting to look into larger networks to see if two different but stable network types of the same size can be found under certain social conditions. The discovery would enable us conduct comparative studies of different social structures.

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