Poverty and Random Walks: Simple Mechanisms Underlying Human Development

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Abstract

1 Introduction

The stubborn persistence of poverty is perhaps the central puzzle of development economics. Almost all theories of poverty reduction revolve around the idea of convergence: the proposition that, because the relationship between capital stocks and income (i.e., the production function) is convex and monotonically decreasing in slope, the poor should eventually "catch up" to the rich. A great deal of the development economics literature is devoted to exploring why in fact this does not happen in the real world. Classically, explanations have focused on differences in savings rates, population growth, and technological change [7]; conditioning on these variables, convergence should hold. Later theorists, while acknowledging the importance of these forces, emphasized the role of imperfections in credit, insurance, and information markets as well as the influence of institutions—durable political and social norms that shape economic behavior [6].Together, imperfect markets and institutions can deform the production function away from the assumed characteristics of convexity and decreasing slope. Even more recently, a growing literature argues for the existence of inherent non-convexities in the production function that lead to "poverty traps" [1]. While the empirical data for poverty traps is to date thin [5], researchers have been encouraged by the intuitive appeal and sheer elegance of the theoretical models.

In this paper, we explore yet simpler mechanisms for the persistence of poverty, in particular the effects of random chance and variation in rates of return to existing wealth operating over empirically relevant time frames. We run a series of agent-based simulations to investigate these topics, calibrating key agent variables and system parameters to fall within ranges conforming to empirical values from a recently collected dataset of rural Ethiopian households [8].[1]

[1]Our model relies heavily on the simple economic model studying the evolution of money distribution proposed by Liang and Carter (2011). It is an agent-based model in which each agent starts with some

Following this introduction, the second section presents a formal mathematical model for our hypotheses and describes the simulations themselves. The third section presents the results, and the fourth discusses these findings in light of existing policies and programs. The final section concludes by summarizing our work.

2 Model and methodology

In this section, we describe our two different simulations. We first outline some general characteristics common to both simulations and then describe the first simulation formally.

2.1 General characteristics of simulations

The simulations are built around a spatial search game. One hundred and forty agents, representing households, are distributed randomly on a two-dimensional grid with patch dimensions 61x51. Resources are also distributed randomly on exactly one-half of the patches on the grid. At each time step, agents move one patch in a random direction and accumulate wealth when they land on a patch containing resources.

Each simulation is run with two initial wealth distributions. One distribution is based on empirical data taken from 140 households living in a poor and food-insecure area of northern Ethiopia, the Eastern Plateau livelihood zone of Tsaeda Amba district of Tigray state [8]. Panel data on these households was gathered in four rounds from August 2011 to February 2013; data from the final round was used in the present simulations. Wealth was measured as the total value of land, livestock, productive tools, and durable goods, minus net debt (i.e., debt - savings). The second wealth distribution we use is uniform, that is, all agents have the same amount of wealth, corresponding to the mean wealth of the empirical sample, expressed in Ethiopian birr (ETB)—2612 ETB ($346 in current PPP-adjusted USD).[2]

Agents are assigned to a wealth group based on the value of their asset stock. Using classifications based on Vaitla et al. (2012)—which itself builds on previous livelihood profiling work in this area [4]—the wealth cut-offs are: destitute ($<$0 ETB net worth), very poor (0-2499 ETB), poor (between 2500-4999 ETB), middle (between 5000-7499 ETB), and well-off ($\geq$7500 birr). Note that all agents are thus initially classified as poor in the uniform distribution. Table 1 summarizes agent income by wealth class in the empirical

initial wealth. At each time step each agent randomly picks another agent from the population and gives this other agent one dollar. Negative wealth (agents in debt) is not allowed in this model; when an agent runs out of money, it does not take any action. It is proven analytically and numerically that if no agents were initially in poverty, any distribution will evolve into a Gaussian distribution whose mean is fixed and variance increases with time. Eventually, as the left tail of the distribution becomes thicker, and since negative wealth is not allowed, the distribution will become Boltzmann-Gibbs, except for points around the origin.

[2] Using a current exchange rate of 18.9 ETB / 1 USD and an purchasing power adjustment factor of 0.4. The adjusted exchange rate is about 7.54 ETB / 1 USD.

distribution, and Table 2 presents summary statistics. Table 3 shows various poverty indices for the uniform and empirical distributions, including simple headcount ratio (P0), poverty gap ratio (P1), and squared poverty gap ratio (P2). These ratios belong to the Foster-Greer-Thorbecke (FGT) class of poverty measures; P1 and P2 weight severe poverty more heavily [3]. Note that P1 and (especially) P2 are considerably worse in the empirical distribution than in the uniform distribution. Figure 1 shows the histogram of the empirical wealth distribution. The Gini coefficient of this distribution is 0.392, signifying moderately high inequality.

Table 1: Wealth classes of empirical distribution

Wealth Class	% of Empirical Distribution	Mean of Class (ETB)
Destitute	2.1	-763
Very Poor	52.1	1412
Poor	36.4	3263
Middle	4.3	5884
Well-off	5.0	9033

Table 2: Descriptive statistics of empirical distribution

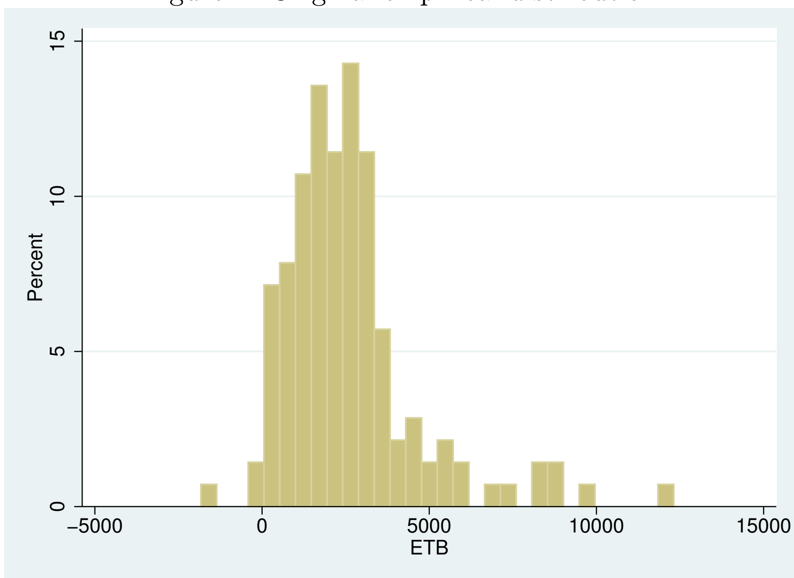
Statistic	Value
Minimum	1828.00
Maximum	12312.50
Mean	2612.30
Standard Deviation	2048.27
Skewness	1.78
Kurtosis	4.77

Table 3: Foster-Greer-Thorbecke poverty measures, original distributions

	P0	P1	P2
Uniform Distribution	1.000	0.478	0.228
Empirical Distribution	0.907	0.525	0.356

The simulation is run for 1826 time steps, which we conceptualize as days—thus about 5 years. Given that Ethiopian households need to spend a minimum of 10.36 ETB for

Figure 1: Original empirical distribution



subsistence, this amount is imposed as a movement cost and subtracted from each agents wealth at each time step.[3] In the first set of simulations ("Simulation 1: Random Walk"), we set the expected value of movement—the probability of finding a resource multiplied by the wealth gained when finding a resource, minus the movement cost—to zero: this scenario is thus a zero-growth economy. We do the same in the second set of simulations ("Simulation 2: Investment Returns to Wealth"), but add the possibility of investment returns. The agents thus receive a pre-specified return to all non-consumed wealth in each time step. The rates of return are set to vary between the equivalent of 2% and 12% annually (that is, every 365 time steps). At the end of each simulation, agents are classified by their asset wealth status, and the overall distribution is described by various summary statistics.

2.2 Formalization

Our simulations explore the effect of a random walk. As mentioned above, the value of the resource is set to equal a zero growth rate—that is, the expected value of movement for each agent is zero—taking into account the size of the grid and the number of agents. Again, the daily costs incurred by agents are equal to the cost of subsistence at the Ethiopian poverty line, and thus each time-step itself represents daily economic exchange around this subsistence level.

[3]This 10.36 ETB/day figure is derived from the Ethiopian national income poverty line for adults [2].

The formal model can be envisioned as follows. Let $w(t)$ be the wealth of a certain agent at time t. At each time step the agent has to pay the cost of movement c, and gains the resource value R if it happens to be on a patch with a resource. Then the wealth of this agent after one time step τ is given by $w(t+\tau) = w(t)+l(t)$, where $l(t)$ is a random variable with two possible values. $l(t)$ has a probability $(1-P_0)$ of being $-c$ and a probability P_0 of being $(R-c)$. Note that P_0 is the resource percentage of the grid. The first 3 moments of $l(t)$ are:

$$\begin{aligned}
\int P(l)dl &= 1 && (1)\\
\int lP(l)dl &= P_0(R-c) + (1-P_0)(-c) \\
&= P_0R - c && (2)\\
\int l^2P(l)dl &= P_0(R-c)^2 + (1-P_0)(-c)^2 \\
&= c^2 - 2P_0Rc + P_0R^2 && (3)
\end{aligned}$$

Let $\rho(w,t)$ be the wealth distribution at time t. The wealth distribution at $t+\tau$ is given by:

$$\begin{aligned}
\rho(w,t+\tau) &= \int \rho(w-l,t)P(l)dl \\
&= \int \rho(w,t)P(l)dl - \int \frac{\partial \rho}{\partial w} lP(l)dl + \frac{1}{2}\int \frac{\partial^2 \rho}{\partial w^2} l^2P(l)dl \\
&= \rho(w,t) - \frac{\partial \rho}{\partial w}(P_0R-c) + \frac{1}{2}\frac{\partial^2 \rho}{\partial w^2}(c^2 - 2P_0Rc + P_0R^2) && (4)
\end{aligned}$$

Consider the Taylor expansion of $\rho(w,t+\tau)$ about t:

$$\rho(w,t+\tau) - \rho(w,t) = \frac{\partial \rho}{\partial t}\tau \qquad (5)$$

Therefore,

$$\frac{\partial \rho}{\partial t} = -\frac{(P_0R-c)}{\tau}\frac{\partial \rho}{\partial w} + \frac{(c^2 - 2P_0Rc + P_0R^2)}{2\tau}\frac{\partial^2 \rho}{\partial w^2} \qquad (6)$$

For simplicity, let $A = -\frac{(P_0R-c)}{\tau}$ and $B = \frac{(c^2-2P_0Rc+P_0R^2)}{2\tau}$. Then the above equation becomes:

$$\frac{\partial \rho}{\partial t} = A\frac{\partial \rho}{\partial w} + B\frac{\partial^2 \rho}{\partial w^2} \qquad (7)$$

To solve the equation above we can start by substituting $\rho(w,t) = \rho_k e^{ikw}$. This gives:

$$\begin{aligned} \frac{\partial \rho_k}{\partial t} e^{ikw} &= Aik\rho_k e^{ikw} - Bk^2 \rho_k e^{ikw} \\ \frac{\partial \rho_k}{\partial t} &= (Aik - Bk^2)\rho_k \\ \rho_k(t) &= \rho_k(0) e^{(Aik - Bk^2)} \end{aligned}$$

Using Fourier Transforms the solution of the initial value problem is:

$$\rho_k(0) = \int_{-\infty}^{\infty} \rho(w,0) e^{-ikw} dw \tag{8}$$

$$\rho(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho_k(0) e^{(Aik - Bk^2)} e^{ikw} dk \tag{9}$$

Now let us solve for the particular case where the initial wealth distribution is uniform, i.e., all agents has the same initial wealth w_0. Uniform wealth distribution is given by the Dirac delta function:

$$\rho(w,0) = \delta(w - w_0) \tag{10}$$

Using equation(8) we find:

$$\rho_k(0) = e^{-ikw_0} \tag{11}$$

Using equation (9)

$$\begin{aligned} \rho(x,t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ikw_0} e^{(Aik - Bk^2)} e^{ikw} dk \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{-\left(\sqrt{Bt}k - \frac{i(At + w - w_0)}{2\sqrt{Bt}}\right)^2 - \frac{(At + w - w_0)^2}{4Bt}} \\ &= \frac{e^{-\frac{(At + w - w_0)^2}{4Bt}}}{\sqrt{4\pi Bt}} \end{aligned} \tag{12}$$

This a normal distribution with

$$mean = w_0 - At$$

and standard deviation

$$\sigma = \sqrt{2Bt}$$

In all of the simulations run in this paper, we set P_0 to 0.5 and vary R to obtain a zero growth rate. Simulation 2 builds off of the basic formalization above to introduce investment returns to wealth.

3 Results

3.1 Simulation 1: Random Walk

Simulation 1 was run for 1826 time-steps (5 “years” in the model) in a zero-growth scenario: the parameter values of P_0 and R were set such that the change in wealth for all agents at each time step was zero. Simulations were run 500 times for each of the two initial (uniform and empirical) wealth distributions. The results of Simulation 1 are shown in Tables 4 and 5 below. The latter shows the FGT poverty measures.

Table 4: Results of Simulation 1

	Org. Uniform	After Random Walk	Org. Empirical	After Random Walk
Destitute (%)	0.0	0.0	2.1	3.5
Very Poor (%)	0.0	40.1	52.1	52.5
Poor (%)	100.0	59.9	36.4	34.0
Middle (%)	0.0	0.0	4.3	5.3
Well-Off (%)	0.0	0.0	5.0	4.7
Mean	2612.3	2619.2	2612.3	2618.3
St.Dev.	0.0	455.0	2048.3	2097.0
Gini	0.0	0.098	0.392	0.405
Skew	-	0.012	1.764	1.660
Kurtosis	-	-0.050	7.559	4.118

Table 5: FGT measures, End of Simulation 1

	P0	P1	P2
Uniform Distribution	1.000	0.476	0.235
Empirical Distribution	0.900	0.526	0.362

The key finding here is that, in the uniform distribution, the Gini coefficient goes from perfect equality to nearly 0.1, a significant increase over a time period of just five years of daily subsistence-level transactions. In fact random chance operating on daily transactions rapidly transforms an initially uniform wealth distribution to one that is normal 95.4% of the time—the number of replications that pass the Shapiro-Wilk test for normality ($p > 0.05$). This is the dynamic that drives the slight increase in the squared poverty gap (P2) rate.

The empirical distribution remains non-normal; however, the random walk influences the shape of the distribution somewhat. Although the distribution remains strongly right-skewed after five “years”, as indicated by the positive skewness value, over time a reduction in the skew occurs—indicating a movement towards normality. The highly leptokurtic initial distribution, with agent wealth concentrated around the mean, is significantly reduced by the end of the random walk.

3.2 Simulation 2: Investment Returns to Wealth

Uniform distribution

Tables 6 and 7, as well as Figures 2, 3, and 4, present the major results for Simulation 2, starting from a uniform distribution of wealth. It is evident that increasing interest rates have an effect on poverty, but perhaps less strongly than expected. Given the level of existing poverty in our Ethiopian dataset, even annual interest rates as high as 10% over a 5-year period succeed only in pulling less than one-tenth of the population over the poverty line. However, there does appear to be a strong non-linearity between 10% and 12% interest. There are some differences with respect to which poverty measure is used, as shown in Figure 4. Headcount reduction accelerates after 8% annual interest, while the poverty gap and squared poverty gap decelerate. The spread of wealth in the distribution increases significantly as interest rates rise, and the Gini falls slightly.

Table 6: Results of Simulation 2, uniform distribution, varying interest rate

	Org.	2%	4%	6%	8%	10%	12%
Destitute (%)	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Very Poor (%)	0.00	20.43	8.91	2.92	0.80	0.00	0.01
Poor (%)	100.00	79.57	91.07	96.83	97.14	91.01	73.31
Middle (%)	0.00	0.00	0.02	0.24	2.06	8.89	26.67
Well-off (%)	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Mean	2612.3	2898.6	3186.7	3506.0	3849.2	4210.4	4611.2
SD	0.00	67.7	73.8	75.6	72.5	79.8	85.2
Gini	0.00	0.093	0.089	0.085	0.082	0.078	0.076
Skewness	-	0.150	-0.048	0.046	0.023	0.088	0.365
Kurtosis	-	-0.450	0.487	0.151	-0.089	-0.225	-0.259

Table 7: FGT measures, Simulation 2, uniform distribution

Interest rate	P0	P1	P2
2%	1.000	0.420	0.186
4%	1.000	0.363	0.141
6%	0.998	0.299	0.100
8%	0.979	0.231	0.066
10%	0.911	0.163	0.038
12%	0.745	0.122	0.044

Figure 2: Wealth classes, uniform distribution, varying interest rate

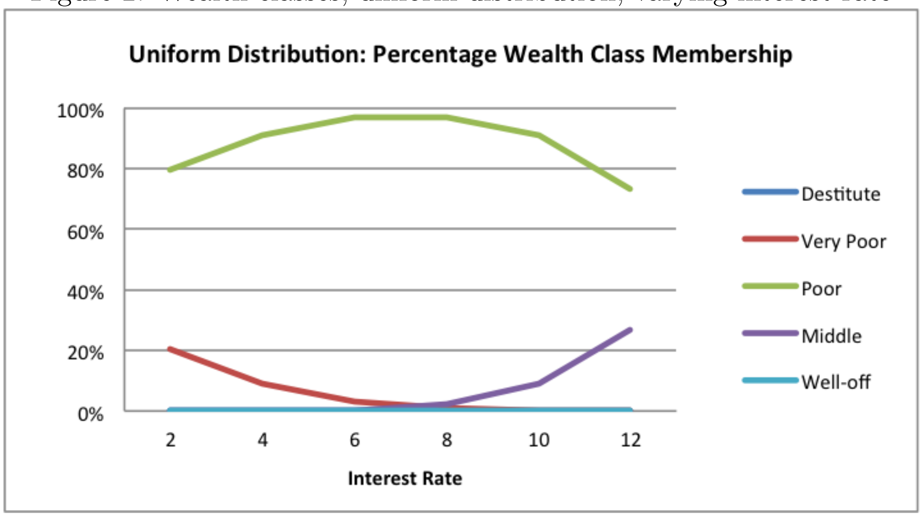


Figure 3: SD and Gini, uniform distribution, varying interest rate

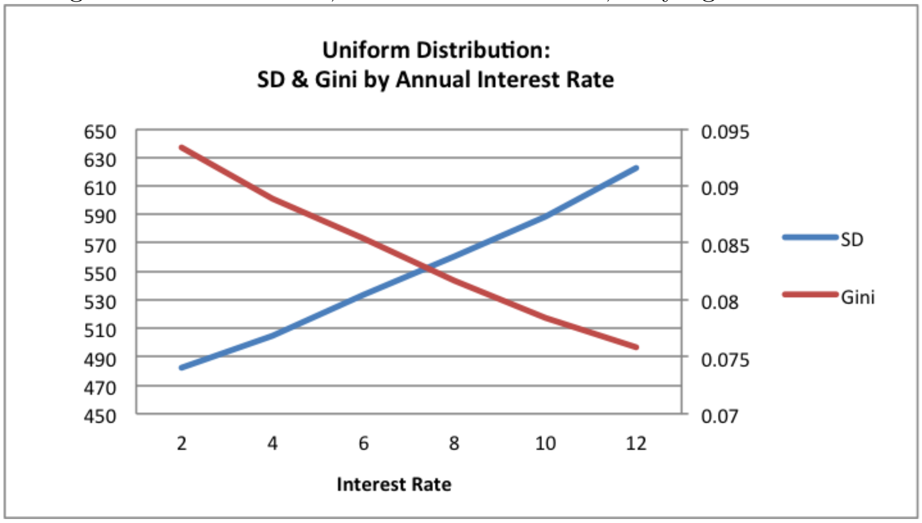
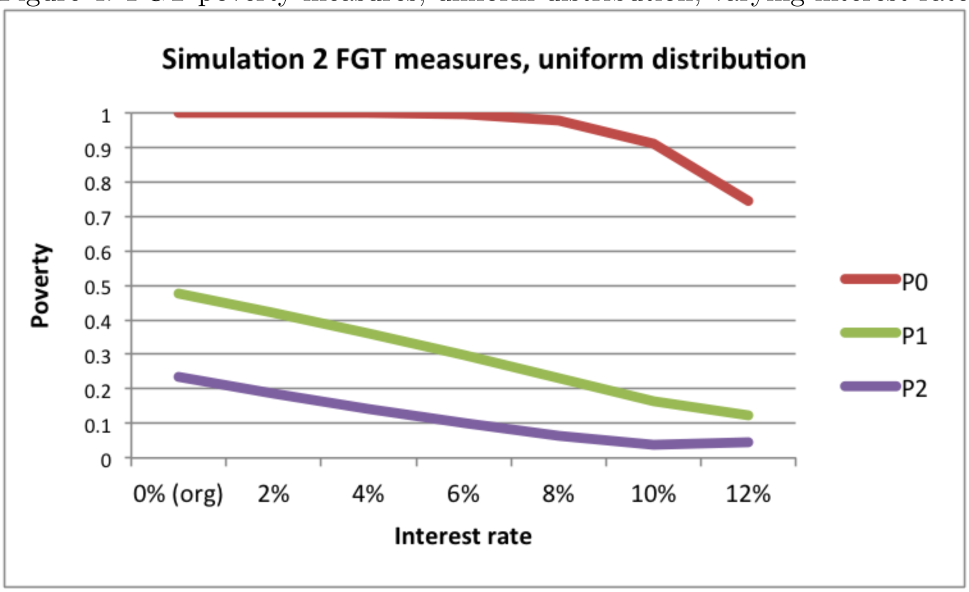


Figure 4: FGT poverty measures, uniform distribution, varying interest rate



Empirical distribution

Tables 8 and 9 and Figures 5, 6, and 7 show Simulation 2 results using the empirical wealth distribution observed in the study communities of northern Ethiopia. Again, the effect of interest rates on poverty reduction is weaker than expected. Even at an extremely high 12% annual interest rate, nearly two-thirds of the population remains poor at the end of 5 years. Improvement in headcount poverty occurs much more rapidly than in the poverty gap and squared poverty gap measures. However, there does appear to be a strong non-linearity between 10% and 12% interest. There are some differences with respect to which poverty measure is used, as evidenced in Figure 7. The standard deviation increases linearly as interest rates increase, although the Gini coefficient is only weakly affected.

Table 8: Results of Simulation 2, empirical distribution with varying interest rate

	Org.	2%	4%	6%	8%	10%	12%
Destitute (%)	2.1	3.14	3.34	2.99	3.54	3.06	2.83
Very Poor (%)	52.1	46.74	40.50	35.83	31.06	27.10	24.09
Poor (%)	36.4	37.64	41.47	42.66	42.46	41.56	37.99
Middle (%)	4.3	6.97	8.39	10.89	13.46	16.74	21.37
Well-off (%)	5.0	5.50	6.30	7.64	9.49	11.54	13.73
Mean	2612.3	2890.2	3172.8	3502.6	3846.6	4211.8	4620.0
SD	2048.3	2266.6	2487.1	2750.1	3031.3	3307.4	3612.2
Gini	0.392	0.404	0.403	0.402	0.404	0.402	0.399
Skewness	1.78	1.80	1.78	1.82	1.77	1.77	1.78
Kurtosis	4.77	4.77	4.75	4.93	4.69	4.73	4.78

Table 9: FGT measures, Simulation 2, empirical distribution

Interest rate	P0	P1	P2
2%	0.875	0.487	0.330
4%	0.853	0.450	0.301
6%	0.814	0.410	0.271
8%	0.770	0.373	0.247
10%	0.717	0.336	0.224
12%	0.649	0.301	.203

Figure 5: Wealth classes, empirical distribution, varying interest rate

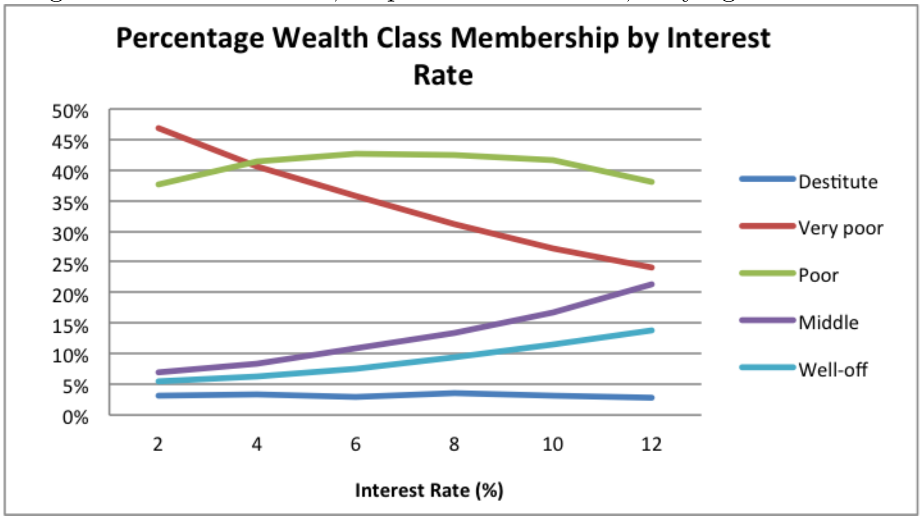


Figure 6: SD and Gini, empirical distribution, varying interest rate

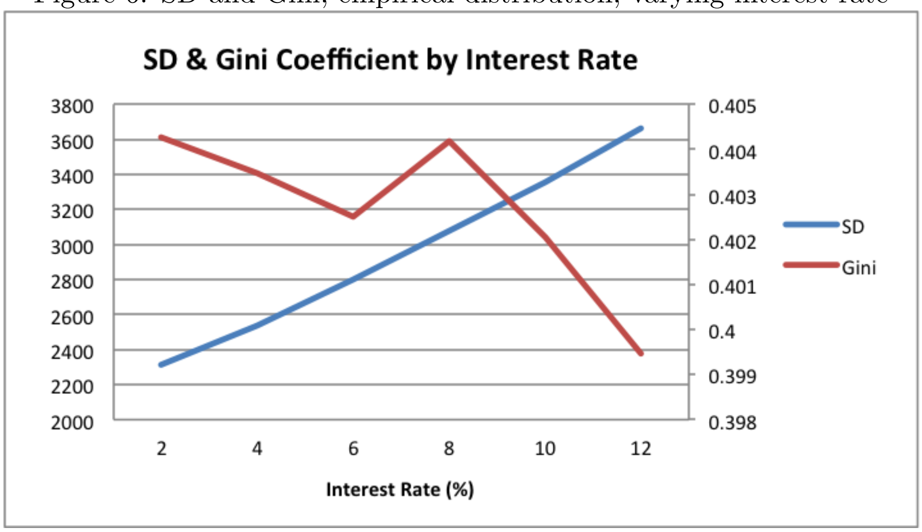
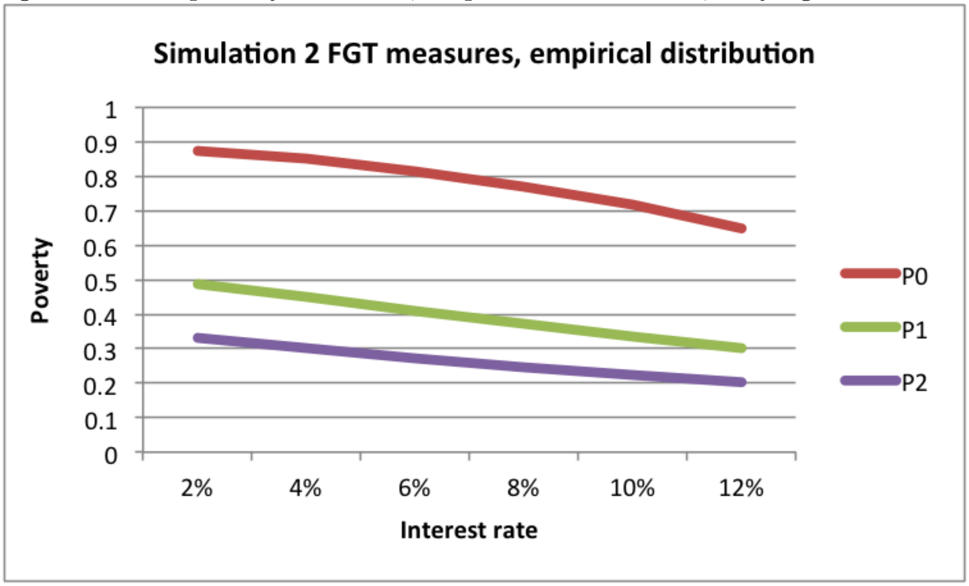


Figure 7: FGT poverty measures, empirical distribution, varying interest rate



4 Discussion

A variety of factors are responsible for the persistence of poverty, ranging from missing markets to poverty traps embedded in the shape of production functions. The simulations in this paper do not challenge the existence of these multiple forces, but rather suggest that a significant proportion of poverty dynamics can be explained by even simpler mechanisms.

Chief among these is the role of stochasticity. We constructed our simulation to model daily subsistence-level transactions among a food-insecure population of northern Ethiopia. While the simulations do abstract considerably—for example, the majority income flows in this area are heavily lumpy, occurring around one to two harvest times a year, although some income is obtained daily or weekly from wage labor and sales of stored produce—they nonetheless suggest that wealth dynamics are affected by random chance, which tends to "push against" equality. A random walk with an expected wealth change of zero at each time step, quickly pushes uniform distributions towards normality over a 5-year period; as seen in Simulation 1; thus even strong redistributive mechanisms will have to confront this countervailing force of random chance. The squared poverty gap increases slightly, highlighting the impact of these dynamics on a small proportion of very unlucky individuals, which ultimately fall into severe poverty.

This has important implications for development efforts. The poverty traps literature has suggested a useful partitioning of poverty into stochastic and structural forms in mapping the relationship between asset stocks and income [1]. Households with asset profiles that map onto non-poverty but are nevertheless poor are said to be in a state of stochastic poverty. In the long-run, accumulation processes should result in these households escaping poverty. Conversely, households with asset profiles that map onto poverty but are nevertheless non-poor are stochastically fortunate, and expected to slip back into poverty over time.

However, this notion of stochasticity holds only if the expected net impact of stochastic shocks over time is zero *for each household*. We see in our simulation that, averaged over the entire population, this assumption holds: the mean wealth is (almost) unchanged. But some households experience a continuous run of "bad luck", and if we are most concerned with severe poverty—as in the squared poverty gap ratio—this negative effect will outweigh the positive movement of households out of poverty.

Furthermore, it is evident from the findings of Simulation 2, that among quite poor populations like those residing in the northern Ethiopia study area, even very high rates of return to invested wealth only have modest effects on poverty reduction; random walk effects will tend to mute potential gains for many poor households. This suggests that development programs such as the Productive Safety Nets Program (PSNP) in Ethiopia, one of the world's largest workfare initiatives, may have to reconsider what are presently unrealistic time frames for households to "graduate" from poverty, particular in the context of volatile market forces that increase investment risk and amplify stochastic shocks and the impacts of random chance.

5 Conclusion

This paper has explored several simple mechanisms by which wealth distributions change, and their implications for welfare of a population living at subsistence level. We have highlighted the power of random chance in pushing already poor populations deeper into poverty or decelerating economic growth, with potentially disastrous consequences on present health, nutrition, and possibilities for future well-being. Policymakers should consider not only predictable shocks in the design of safety nets, but also take into account the role of stochasticity even in daily economic interactions. Over time, this quotidian stochasticity can seriously impair the ability of households to build assets, especially when rates of return to livelihood investments are low.

References

[1] Michael R. Carter and Christopher B. Barrett. The economics of poverty traps and persistent poverty: An asset-based approach. *The Journal of Development Studies*, 42(2):178–199, 2006.

[2] Development Planning and Research Directorate (Ministry of Finance and Economic Development, Government of Ethiopia). Ethiopia's progress towards eradicating

poverty: An interim report on poverty analysis study (2010/11). Technical report, Addis Ababa, Ethiopia, 2012.

[3] James E. Foster, Joel Greer, and Erik Thorbecke. The Foster-Greer-Thorbecke (FGT) Poverty Measures: Twenty-Five Years Later. 2012.

[4] Livelihoods Integration Unit (Ministry of Agriculture and Rural Development, Government of Ethiopia). Tigray Region livelihood zone profiles. Technical report, Addis Ababa, Ethiopia, 2009.

[5] Andy McKay and Emilie Perge. How strong is the evidence for poverty traps? A multicountry assessment. *Journal of Development Studies*, 49(7):877–897, July 2013.

[6] Douglass C. North. *Institutions, institutional change, and economic performance*. Cambridge University Press, Cambridge, 1990.

[7] Robert M. Solow. A contribution to the theory of economic growth. *The Quarterly Journal of Economics*, 70(1):65–94, 1956.

[8] Bapu Vaitla, Girmay Tesfay, Megan Rounseville, and Daniel Maxwell. Resilience and livelihoods change in Tigray, Ethiopia. Technical report, Feinstein International Center, Boston, MA, 2012.