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If we are working with random graphs, it has been shown that there is an edge density level at which the probability of there being a 3-coloring of the nodes of the graph becomes zero. [Stated another way, no 3-coloring can exist of any random graph once a certain fraction of the edges possible in the graph are present.] This phase transition marks the edge of solvability for this class of problem, but there remain questions about the relative difficulty of finding solutions. It seems probable that many problems in the solvable range will require a great deal of computation to solve. Another conjecture is that the difficulty of finding these solutions will scale with the density of the random graphs involved. In order to investigate these problems, we will perform computational experiments measuring the average number of changes that are needed to correct the coloring of random graphs of various sizes and densities. Rather than working with the original discrete formulation of the coloring problem, we will instead focus on the case in which each node is associated with a real number from 0 to 1, with the restriction that any two neighboring nodes must be at least 1/3 (modulo 1) away from each other. This change to the original problem allows us to focus on changes which only affect one neighbor at a time, as opposed to the discrete formulation, in which changing the color of one node can affect every one of its neighbors simultaneously.

The problem of modeling the motion of hard spheres in close proximity to each other has many applications in physics. The idea is that there are some systems which we can profitably think of as being composed of inelastic homogenous spheres, whose interacting motions are then simulated to study various features of the system in question. Many fundamental problems in statistical mechanics are studied using this simplified model, making the difficulty of computationally simulating these interactions a serious barrier to our further understanding. Our approach to graph coloring is intended to operate in the same fashion as the event-chain algorithm for performing such simulations, and our experiments should uncover results which are useful to the design and application of such algorithms.