Adaptation And Learning In Blotto Games

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Abstract

When Game Theory began to take off, the games that propelled it were The Prisoner's Dilemma and Colonel Blotto. The Prisoner's Dilemma, perhaps the most popular of games, thrived through much of its history, because of its broad applications. Its lesser known cousin, Blotto, sadly did not fare so well. In this paper, I show that equilibria can be found computationally by playing a large number of games. To do this, continuing with the military theme, I construct a model of a repeating Colonel Blotto game consisting of two players who are given troops to allocate across a given number of battlefronts based on predetermined strategies. In each game, the colonels add weight to the strategies they use and eventually they converge on optimal strategies. This research's goal is to show that optimal strategies can be found computationally using a simple learning technique.

1 Introduction

[1] Game theory allows us to formalize the study of strategic interactions by using the power of mathematics, economics, and rational choice. With political science in particular, game theory can help understand the imbalance of power and what decisions actors strategically make to either uphold or upset it. It is with this in mind that I study the Colonel Blotto game. Blotto is a game that from the mid twentieth century until recently[6, 1, 2, 7], lagged in popularity behind its better known cousin, the Prisoner's Dilemma. However, since Roberson[6], there has been a renewed interest in Blotto games with applications to situations involving the military, campaign finance, business RD allocation, and even sports[2].

Blotto is a class of two-person zero-sum games where colonels are tasked to simultaneously allocate troops over a number of battlefronts. The game, first proposed by Borel(1921) and later refined by Gross and Wagner(1950), has several applications to military, business, and political problems that arise in the real world. However, as the number of troops or battlefronts increase, the game's complexity also increases which makes equilibria harder to find. In this paper, I show that equilibria can be found computationally by playing a large number of games. To do this, continuing with the military theme, I construct a model of a repeating Blotto game consisting of two colonels who are given troops to allocate across a given number of battlefronts based on a predetermined set of strategies. In each game, that the colonels win they add weight to the strategies that they use and eventually converge on best response strategies. This research shows that using a simple learning technique, optimal strategies can be found computationally for large games that are computationally intractable for analytical analysis.

Though first proposed in 1921, Blotto games

[1]Special thanks to Nathan Collins for his advice and help.

were largely ignored by the general economics, political science, and mathematics communities, although they were dabbled with by a handful of mathematicians. The game received its name and generally accepted context from a 1950 paper by [3], who tasked their colonel with finding an optimum distribution across n battlefields to defeat an enemy with equal forces. In their game, the colonel knows that if he allocates more troops to a given battlefield, he wins, but if he allocates less then he loses. Neither he nor his opponent know where their enemy will allocate troops so they must distribute in such a way as to maximize the number of battlefields they expect to win.

2 Blotto Games

2.1 The Classic Blotto Game

Let a Colonel Blotto game be a two-player game between players A and B. In the game, players only care about winning more battlefronts m than their opponents. A player wins a particular battlefront when he commits more troops n to the front than his opponent. Ties are decided with a coin-flip. I now define some terms that are important to the study of Blotto Games.

Definition (Strategy Set) For each front i, a colonel X can allocate a discrete number of troops across a battlefield such that $X_S = (X_1, X_2, ..., X_m)$ with

$$\sum_{i=1}^{m} x_i = 1, x_i \in [0, 1]$$

where x_i represents the fraction of the budget allocated to front i.

Definition (Payoffs) Payoffs in the traditional Blotto Games are calculated as auction games[3]. Where a player's payoffs are,

$$\sum_{i=1}^{m} sgn(x_i - y_i);$$

where the function,

$$sgn(X) = \begin{cases} 1, & \text{if } X > 0, \\ 0, & \text{if } X = 0, \\ -1, & \text{if } X < 0, \end{cases}$$

It is always assumed that $m > 2$; otherwise the game will always end in a tie. Now that I have the structure of a basic Blotto game designed. I can play a game.

Example Let each player have $n = 6$ troops to allocate across $m = 3$ fronts with a marginal uniform distribution of the allocation sitting on the interval $[\frac{1}{m}, 1]$. Their allocations are as follows.

Player 1(A)	2	2	2
Player 2(B)	1	1	4
Winner	A	A	B

In the game above, $n = 6$ for both players and $A_S = (2, 2, 2)$ against $B_S = (1, 1, 4)$. Clearly A is the winner in this contest. As a matter of fact, when the proportion of troops allocated to each front falls within the interval of $[\frac{1}{m}, 1]$, A_S is the pure strategy for this particular rendition of Blotto games. As the number of troops and the number of fonts increase however, pure strategies cease to exist in the symmetric case. Colonel Blotto does, however, have mixed strategy equilibrium for $m > 3$ in which marginal distributions are uniform on $[0, \frac{2}{m}]$ along all fronts[3] .

Also of note with traditional Blotto games is an observation[2], where I let $n = \frac{m^2 - m}{2}$, that permutations can be mixed to allocate troops uniformly on $[0, \frac{2n}{m}]$.[2] This will be revisited later on in the model

[2] More on this can be found in Golman and Page(2009)

2.2 Asymmetry And Contest Success Functions

Most of the work done with Blotto has been done with symmetric resources to allocate for both players. Recent work[1] has tackled cases of Blotto where players do not have symmetric resources. To do this, the researchers used a "lottery" contest success function in which the probability of winning a battlefield equals the ratio of a player's resource allocation to the sum of the players' resource allocations in that battlefield. An extension to the lottery CSF brings the analysis closer to the form of more general CSF's of the ratio form.

Definition (Ratio CSF) Players allocate their forces across a battlefield according to some marginal univariate distribution. The probability that player i wins a single front is calculated as a function of the two levels of expenditure X_i and X_j is,

$$\frac{X_i^\kappa}{X_i^\kappa + X_j^\kappa}$$

where $0 < \kappa$.

Here I demonstrate a ratio game with asymmetric resources.

Example Let $A_s = (0, 1, 2, 3, 4)$ and $B_s = (1, 2, 3, 0, 0)$. they are fighting across a battlefield where $m = 5$ and the n=10

Player 1(A)	2	2	2	2	2
Player 2(B)	1	2	3	0	0
Probabilities$A$	.6	.5	.4	1.	1.

At this point a number chosen from a random uniform distribution on the interval of [0,1], if it lands above player 1's percentage of winning, he loses, if it lands under his percentage, he wins. This game allows interesting combinations, however there exists a flaw. When the ratio form applies each side will surely always commit some resources to a given front[4]. If peace is defined by the condition that $X_i = X_j = 0$, then peace can never occur as an equilibrium with the ratio success function. Suppose $X_i = X_j = 0$ is indeed the case, under the Cournot assumption either player would be motivated to defect, since even the smallest commitment of resources makes the defector's relative success jump from 50% to 100%[4]. Chowdhury and his colleagues reflected this in their experimental results[1]. To account for this kung-fu treachery, I use a new CSF proposed by Hwang in 2009[**?**]. It is an integration of the ratio success function and another function, the difference success[3] function. It uses the two as limits. With that in mind, I define our new function.

Definition (Integrated Contest Success Function) I call $u(X_i, X_j; \rho)$ and write it out as,

$$u(X_i, X_j; \rho) = \frac{\exp(\kappa \frac{1}{1-\rho} X_i^{1-\rho})}{\exp(\kappa \frac{1}{1-\rho} X_i^{1-\rho}) + \exp(\kappa \frac{1}{1-\rho} X_j^{1-\rho})}$$

Where κ is a "mass effect parameter upon the shape of X_i's contest success function"[4] and measures the slope of the contest success function in an evenly balanced match. ρ is the elasticity of augmentation, which is a normalized percentage increase in marginal rate of augmentation[5]. $X_i = X_j = 0.5$ holds throughout regardless of the value of κ or ρ.

When ρ is low, I expect side 1 would need to augment its resources by a smaller amount to keep up the same probability of success. When $\rho = 0$ I expect the integrated CSF to take the shape of the difference form, and as $\rho \to 1$, I can expect the CSF to look more like the ratio form. Of course I'm going to continue taking payoffs in the form of probabilities just as I did with the ratio function. Now that the game is well defined, I are going inspect whether a computer can find best response strategies.

[3] the difference success function by itself does not admit interior pure strategy Nash equilibria. For more information on this see [4, **?**]

3 The Model

To test our hypothesis, an agent-based model was constructed. In the model, two agents play Blotto Games repeatedly for a specified time t. As the games go on, player A uses a weighted choice technique to assign weights to the various strategies its using. Strategies are kept in a playbook by each agent to draw upon. As weights accumulate and diminish according to the success of the strategy against other strategies, A eventually settles on a set of strategies that are best responses to generally anything its opponent B throws at it. With this in mind, I consider optimal approaches to learning. However in the test runs, the model did not output best response strategies. In the model, if a player has a significant advantage he doesn't settle on a pure strategy but instead continues trying new strategies; likewise disadvantaged players tend to settle on one or two extremely sub optimal strategies, which are not equilibria. This is the opposite of what should happen in the model. Pure strategies should exist in asymmetric games, as well as a small subset of symmetric games. This was investigated and the most likely culprit is not the selection mechanism exclusively, but also the strategy space. Although the strategies the agents are using from the playbook are optimal locally within the context of their play books, the strategies are not optimal globally. I found that while trying to run the simulation to find well known equilibria[2]. More work needs to be done to make sure the algorithm can find correct equilibria. Currently a new way of adopting strategies is being researched, and rather than a playbook, agents will be able to simply create an optimal strategy given the situations its been involved in. An objective function for strategy selection has been talked about in the literature[2, 7] and even implemented[7]. Following their example, I will refine our strategy generation and and selection technique to better choose mixed and pure strategy equilibria.

4 Conclusion

In this paper I have proposed a way to find a robust strategy generation and selection mechanism, and future work will be to that effect. Blotto games have only recently began to shed their mystery and reveal the intricacies that have been left hidden for all of these years. Research is currently ongoing.

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