INTRADAY DYNAMICS AND BUY SELL ASYMMETRIES OF FINANCIAL MARKETS

AARON MILLER
UNIVERSITY OF NEW MEXICO, ALBUQUERQUE, NM 87131

Abstract. Financial markets exhibit strong stochastic behavior, making predictions on price change difficult. Observing the market behavior on transaction level can show how prices change on extremely short time scales. I analyze the impact of individual trades as well as market response to buy and sell transactions. I will also investigate the autocorrelation of trades in the market, showing an increasing correlation for longer times within a day. Finally, I observe what impact the tick size has on market dynamics, showing smaller tick size allows the price to change more often, but in smaller average steps.

1. Introduction

Stock markets have been studied for many years as systems exhibiting complex behavior. With huge numbers of participants, each subject to the rationalities and irrationalities of human behavior, prices behave stochastically, even before electronic trading became commonplace[1]. Expectations are formed on the expectations of others, creating a network of constantly changing and often self-referential expectations [2].

Financial markets allow for rapid exchanges of equity based on participants perceptions of stock value verses the current price and trends in the price. According to the efficient market hypothesis (EMH), investors cannot consistently out perform the market average returns unless higher risk investments are chosen. There is evidence that the EMH holds in financial markets and that newly available information is the driving force behind prices [3]. However, more recently Shiller observed prices change too quickly to be completely dependent on new information [4]. Maldarella and Pareschi present a kinetic model where agents base their opinion on the opinions of others, where expectations diffuse through the population [5]. Therefore, a combination of these factors dictates the stock price.

Markets can be studied at a plethora of different time scales. Yearly indices are better for examining long, historical trends, to account for seasonal variations. Quarterly reports are widely used by firms to describe earning and employment performance. Daily indices, such opening and closing prices, can reflect the effect of newly available information on perceived asset value.

Since the true impact of external shocks are difficult to measure, longer time scales can be muddled by these events. For example, in the days immediately after the September 11th attacks on the World Trade Center, the London Stock Exchange experienced a sharp dip in prices. By the end of the next week, however, prices seemed to have stabilized. It would be short-sighted to claim that this quick price shift was the total effect of the attacks, but summarizing the long term effects is a much more convoluted issue. Furthermore, such events can cause result in extreme returns, which often violate the average day-to-day behavior of stock markets, as Longin observed [6].

Financial markets can exhibit extreme levels of volatility in short time intervals, hinting that price fluctuations can occur independently from newly available information. Instead, market velocity can be self fulfilling, as traders can attempt to minimize losses (or the potential losses incurred by not buying) by selling shares when the price is headed down (or buying when it is moving up). Therefore, studying the mechanical effects of transactions can illuminate what causes sudden price changes and reveal how participants react to the actions of other market participants.

1.1. **Order Types.** A broker has two typical tools he or she can use to exchange shares of stock: limit orders and markets orders. A market order is executed at the time the order is placed, resulting in an immediate transaction at the best available price. The spread is defined as the gap between the best ask (lowest price at which shares can be bought at a given time) and best bid prices (the highest price at which shares can be sold at a given time). Another important attribute of a stock is the tick size, the smallest movement the stock price can show. Tick sizes vary by stocks, depending on the price. A limit order is placed into the limit order book to be executed when the the limit order price can be matched with an opposing market order. If a limit order to buy can be immediately matched with a limit order to sell, the transaction is fulfilled. Typically, a market order is characteristic of an impatient trader, while a limit order delineates a more patient investment strategy [7].

Brokers can choose to place market or limit orders to buy or sell shares. For the purposes of this paper, any order (or fraction of one) that is immediately filled will be referred to as a market order, and any order (or part of one) that is placed in the order book will be deemed a limit order, rather than being classified by the brokers intent.

1.2. **Data.** I use London Stock Exchange data from 2000-2002. I select nine stocks to observe: Astrazeneca (AZN), BHP Billiton (BLT), British Sky Broadcasting Group (BSY), LLoyds Tsb Group (LLOY), Prudential (PRU), Rentokil Initial (RTO), Reuters Group (RTR), Tesco (TSCO), and Vodafone (VOD). The first and last thirty minutes (8:00-8:30 and 16:00-16:30) and the off-hour auctions were excluded, due to the strong volatility observed during these times. Each transaction includes information on the type of transaction, the volume purchased, the time it was executed, the best ask and bid prices, and a unique broker code. Each broker is granted a unique identifying code every month, to maintain anonymity of the broker.

Assuming a zero-intelligence model [8], the number of buy and sell orders should not significantly differ. However, this assumption almost never holds; an observable, extremely statistically significant difference can be observed. Figure 1 shows the distribution of ratio of buy to sell orders during each day in the time interval.

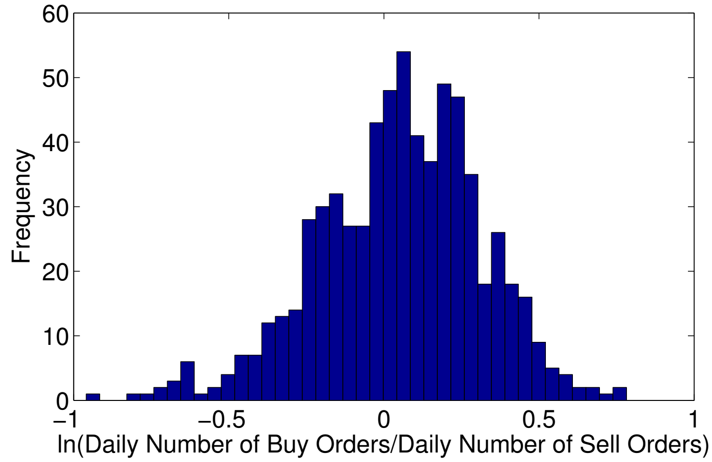

FIGURE 1. Histogram of daily buy/sell ratio for Vodafone

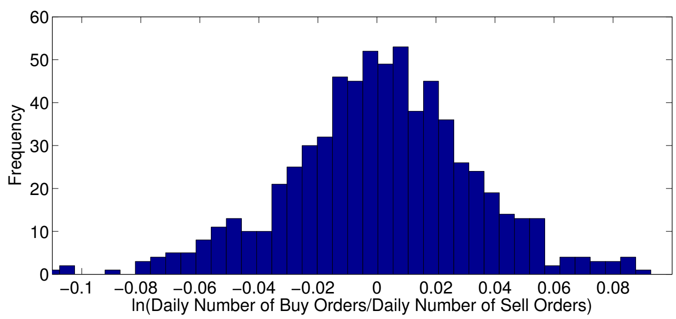

FIGURE 2. Histogram of random set of daily buy/sell ratio, assuming orders have P=.5 to be buy and P=.5 to be sell. Note that right hand outliers are only correspond to a ratio of 11 buy orders to 10 sell orders, while the right hand outliers of Figure 1 correspond to a ratio of about 21 buy orders to 10 sell orders.

2. Impact and Price Response

2.1. **Market impact.** To calculate the price change of a stock, the price is calculated immediately before the transaction (the event used to calclulate this price can be from a limit order, a limit order expiration, or a deleted order), and immediately after the transaction takes place. The price is given by the midpoint m of the best ask and best bid prices, given by:

$$p_t = [a_t + b_t]/2 \tag{1}$$

Where a is the best ask price and b is the best bid price. From this point forward, the word price will actually refer to the natural logarithm of the price, to ensure comparability of price change across stocks with different tick sizes and prices.

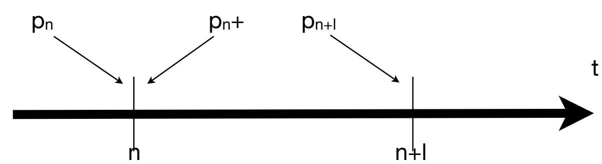

FIGURE 3. Notation used when referencing prices

Figure 3 shows the method for defining price. p_n refers to the price immediately before a transaction is completed, p_{n+} gives the price right after the transaction is complete, and p_{n+l} denotes the price immediately preceding the transaction l trades later.

[11] defines the market impact on the price of a stock i as:

$$r_i = ln[p_{i+}] - ln[p_i] \tag{2}$$

This equation analyzes the mechanical effect of a single transaction. Additionally, I average r_i across many transactions to find the average impact. r can also be considered a function of the volume.The impact function $r_i(v)$ represents the expected change in the price when a transaction of volume v is executed.

$r_i(v)$ follows a power law, scaling as:

$$r_i(v) \sim \alpha v^\beta \tag{3}$$

Using a non-linear least squares regression, the α and β parameters can be estimated. The following table gives a 95% confidence interval for the values.

Stock	β Buy	α Buy	β Sell	α Sell
AZN	(.144-.277)	(2.28×10^{-5} - 7.29×10^{-5})	(.113-.299)	(1.27×10^{-5} - 8.31×10^{-5})
BLT	(.276-.455)	(1.66×10^{-5} - 3.13×10^{-5})	(.199-.432)	(-$.411 \times 10^{-5}$ - 5.72×10^{-5})
BSY	(.0817-.170)	(1.02×10^{-4} - 2.39×10^{-4})	(.0963-.190)	(8.10×10^{-5} - 1.99×10^{-4})
LLOY	(.223-.332)	(1.34×10^{-5} - 4.12×10^{-5})	(.191-.310)	(1.53×10^{-5} - 5.29×10^{-5})
PRU	(.199-.329)	(2.02×10^{-5} - 7.62×10^{-5})	(.118-.335)	(2.54×10^{-6} - 1.25×10^{-4})
RTO	(.236-445)	(-7.63×10^{-7} - 5.41×10^{-5})	(.267-.479)	(-1.09×10^{-6} - 3.95×10^{-5})
RTR	(.169-.260)	(5.96×10^{-5} - 1.45×10^{-4})	(.239-.321)	(3.66×10^{-5} - 8.19×10^{-5})
TSCO	(.224-.412)	(5.79×10^{-5} - 3.11×10^{-5})	(.148-.387)	(-5.39×10^{-6} - 5.36×10^{-5})
VOD	(.293-.365)	(4.92×10^{-6} - 1.27×10^{-5})	(.314-.379)	(4.16×10^{-6} - 9.71×10^{-6})

2.2. **Response Function.** The response function $R(l)$ as defined by [10] is:

$$R(l) = \langle [p_{n+l} - p_n]\epsilon_n \rangle \tag{4}$$

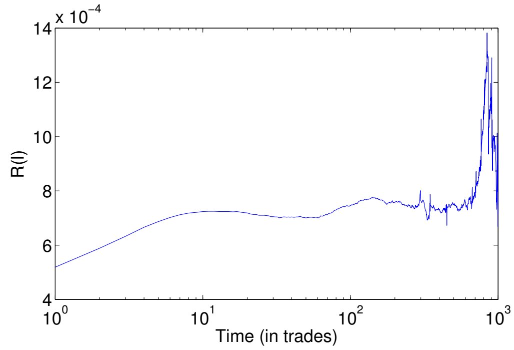

FIGURE 4. Response function for Vodafone in 2002

Where ϵ of is the sign of the nth trade (+1 for a buy transaction, -1 for sell), and p_{n+l} is the price l time later. Time is discritized in this equation; each additional transaction is a step in time.

3. Price Response from Buy and Sell Transactions

To investigate the asymmetries in market response to buy and sell trades, this response function can also be conditioned on the sign of the initial trade, giving:

$$R(l) = \langle [(p_{n+l} - p_n)\epsilon_n | \epsilon_n = 1] \rangle \tag{5}$$

for buy transactions and

$$R(l) = \langle [(p_{n+l} - p_n)\epsilon_n | \epsilon_n = -1] \rangle \tag{6}$$

for sell trades.

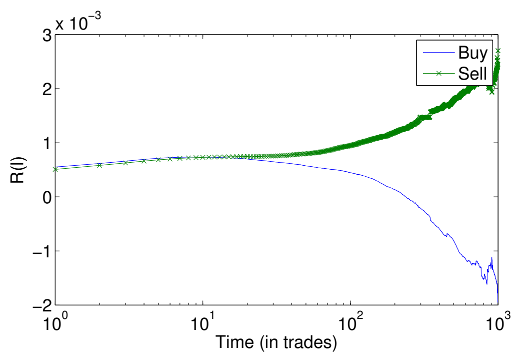

FIGURE 5. Response function for Vodafone in 2002, conditioned on the sign of the initial trade

For smaller values of l, the structure of the curves are very similar. As l grows, the response increases slowly before reaching a peak and then decreasing. For all nine stocks, the buy curve is above the sell curve, once again suggesting that the buy transactions have a greater effect on the price (See Figure 6).For larger values of l, the buy and sell curves begin to diverge from one another, typically between 10 and 100 trades. For each of the 9 stocks in 2002, the sell curve turns upwards and the buy curve moves downward (See Figure 5). Larger values of l tend to be extremely noisy and often deviate into negative values. A negative value for $R(l)$ for a buy trade would indicate that a buy transaction actually decreases the price in the long run, which is inconsistent with the basic mechanics of a financial market.

This divergence into negative values can be explained, however. Each of the nine stocks exhibits a negative price drift over the time interval (2002). For larger values of l, the initial trade must be near the beginning of the day, while l trades into the future will be near the end, since the response is calculated on an intraday basis. Therefore, $R(l)$ captures the average daily price drift and the response functions are not accurate for large l. Vodafone serves as a good stock to observe, due to the large number of trades in a given day.

4. Autocorrelation

The Efficient Market Hypothesis assumes that each actor in the market uses the accessible information perfectly. In reality, this assumption does not hold, due to the relevant information sometimes being deemed as unreliable by those with knowledge of it, or simply lost in the sea of irrelevant information. Instead of relying on information outside the market, some traders assume that other participants are more informed, and base their trades on other participants. This market strategy can be self fulfilling, as uninformed traders might be making decisions based on other uninformed traders, leading to a strong deviation from the real value of a stock.

To measure this effect, the autocorrelation of trades can be measured. [10] and [12] introduce the correlation function:

$$C_0(l) = \langle \epsilon_n \epsilon_{n+l} \rangle - \langle \epsilon_n \rangle^2 \tag{7}$$

As before, ϵ is the sign of the trade, l is time (in transactions after the initial transaction) and the correlation is calculated by averaging the function result across many values of n.

This function is sufficient when buy and sell orders are aggregated together, but a problem arises when buy and sell orders are segregated. The assumption that buy and sell orders occur in equal frequency is flawed. On daily intervals, the ratio of buy orders to sell orders varies greatly, with values significantly different from 1 appearing often. To account for this, the correlations must be calculated using conditional probabilities, namely:

$$P_1(l) = P(\epsilon_{n+l} = 1 | \epsilon_n = 1) - P(\epsilon_n = 1) \tag{8}$$

for buy transactions and

$$P_2(l) = P(\epsilon_{n+l} = -1 | \epsilon_n = -1) - P(\epsilon_n = -1) \tag{9}$$

for sell transactions.

When plotted as a function of l, both functions exhibit decreasing correlations for small values of l. The values of $P_1(1)$ and $P_2(1)$ are fairly similar across all stocks, approximately equal to .1. The correlation then decays to nearly .02, with a minimum between 50 and 100 trades.

For larger values of l, the conditional probabilities begin to increase. This behavior is not observed in the non-conditional probability function:

$$P_3(l) = P(\epsilon_{n+l} = \epsilon_n) \tag{10}$$

This increasing correlation for large values of l is not observed on an interday basis [10] [12], therefore this phenomenon must be the result of intraday dynamics. Further investigation is required to find the cause.

5. Effect of Tick Size

The tick size of a stock, the smallest jump in price that a stock can experience, can have an impact on the dynamics of a market. These tick sizes are not static, as they can be changed if the value of a stock changes enough. A smaller price requires a smaller tick size, to ensure the minimum price change is not too large compared to the stock price.

For the stocks analyzed tick sizes range from one pence (AZN) to one-half pence (LLOY), to one-quarter pence (BLT, RTO, TSCO, VOD). Both RTR and BSY exhibit tick size changes from one to a half pence in 2001, and the tick size of RTR decreases again in 2002 to one-quarter pence.

Analyzing tick size changes is useful, especially considering one can pinpoint the exact moment that the change occurs. After a tick size change, the volume conditioned probability that a transaction induces a change in the midprice increases. That is:

$$P_4(x) = P[p_n \neq p_{n+} | v < x] \tag{11}$$

For large values of x ($x > 100000$), $P_4(x)$ is practically constant, due to the substantially greater number of transactions with volumes of less than 100,000 than more than 100,000. For the intents of this paper, $P_4(100000)$ will be considered the total probability of a price change for a given stock.

Comparing $P_4(x)$ from the two weeks before a tick size change and the two weeks after shows a substantial increase of about 8 percent for BSY(See Figure 9), with RTR showing about a 5.5 percent increase after both tick size changes (See Figure 8). This result mirrors the intuitive expectation that a finer resolution for the price allows it to shift more often.

Conversely, the response function $R(l)$ conditioned on a price change:

$$R(l) = \langle [(p_{n+l} - p_n)\epsilon_n | p_{n+l} \neq p_n] \rangle \tag{12}$$

decreases for RTR with a smaller tick size, meaning that even though the probability of a price change increases, the average size of the change decreases.

However, it is difficult to compare the response functions for periods with different prices, so a scaling method must be used. By plotting the tick price divided by the average price over the time interval, which in this case is the total time over which a specific tick size is used, shows a strong negative correlation for $P_3(x > 100000)$ and a strong positive correlation for $R(1)$ (See Figures 10 and 11). (Note that here the average price does not mean the logarithm of the price, it means the nominal price.)

6. Conclusion

The purpose of this paper was to investigate the intricacies that arise at the microstrucural level in financial markets.

The asymmetries between buy and sell transactions were not as large as I originally thought. The values for β and α in equation (3) differ, but not by any predictable amount. Equations (5) gives a slightly higher value than (6) for small values of l across all nine stocks for 2002, the opposite of the dominance of the sell curve due to price drift for large l values, suggesting that buy transactions have a slightly larger impact on the price, at least in the short term. This irregularity persists even when equation (12) is applied, providing more evidence that buy transactions do have a stronger effect.

The upswing that occurs for large values of l in equations (8) and (9) pervades each of the nine stocks, hinting that it is not just a statistical irregularity. I investigated the possibility that market behavior (in terms of number of buy and sell transactions) is similar during the beginning and ending of the day during high frequency trading days, but there was no strong evidence to suggest it to be true. Further exploration of the topic might lead to some interesting discoveries.

Tick size changes appear to cause a distinct change in market dynamics. Prices move more often, but with smaller changes. Further analysis suggests that the ratio of tick size to price is more revealing, in fact correlating with response and the probability of a price change.

Human behavior is very difficult to predict, due to the large number of factors that influence each decision. Decisions made by machines, on the other hand, must follow very well defined logic. Therefore, the increasing number of high-frequency automatic traders could add more regularity to market microstructure. In fact, Smith [13] observed that average size of trades (in shares) has been decreasing and smoothing out in the period of 2002 to 2010, with the Hurst exponent over short time scales (less than 15 minutes) increasing from near .5. Since this paper takes all its data from 2002 and before, newer data might show stronger structure.

References

[1] Fama E F 1965 The Behavior of Stock-Market Prices *The Journal of Business* **38**, 34

[2] Arthur W B 1995 Complexity in Economic and Financial Markets *Complexity* **1**, 1

[3] Fama E F 1970 Efficient Capital Markets: A Review of Theory and Empirical Work *The Journal of Finance* **25**, 383

[4] Shiller R J 1981 Do Stock Prices move too much to be justified by subsequent changes in dividends? *American Economic Review* **71**, 421

[5] Maldarella D and Pareschi L 2010 Price dynamics in financial markets: a kinetic approach *eprint arXiv* **1007**, 1631

[6] Longin F M 1996 The Asymptotic Distribution of Extreme Stock Market Returns *The Journal of Business* **69**, 383

[7] Smith E 2003 et al Statistical theory of the continuous double auction *Quantitative Finance* **3**, 481

[8] Farmer J D, Patelli P, and Zovko I I 2005 The predictive power of zero intelligence in financial markets *Proceedings of the National Academy of Sciences in the United States of America* **102**, 2254

[9] Farmer J D and Lillo F 2004 On the Origin of Power Law Tails in Price Fluctuations *Quantitative Finance* **4**, 7

[10] Bouchaud J P, Gefen Y, Potters M, Wyart M 2008 Fluctuations and response in financial markets: the subtle nature of 'random' price changes *Quantitative Finance* **4**, 176

[11] Lillo F, Farmer J D, and Mantegna R N 2003 Master curve for price impact function *Nature* **421**, 129

[12] Lillo F and Farmer J D 2004 The Long Memory of the Efficient Market *Studies in Nonlinear Dynamics & Econometrics* **8**, 1

[13] Smith R D 2010 Is high-frequency trading inducing changes in market microstructure and dynamics? *eprint arXiv* **1006**.5490

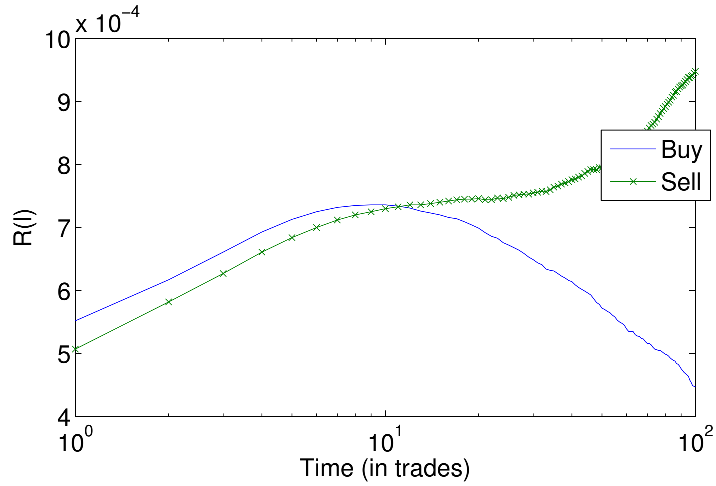

Figure 6. Response function for Vodafone in 2002, conditioned on the sign of the inital trade, zoomed for higher resolution

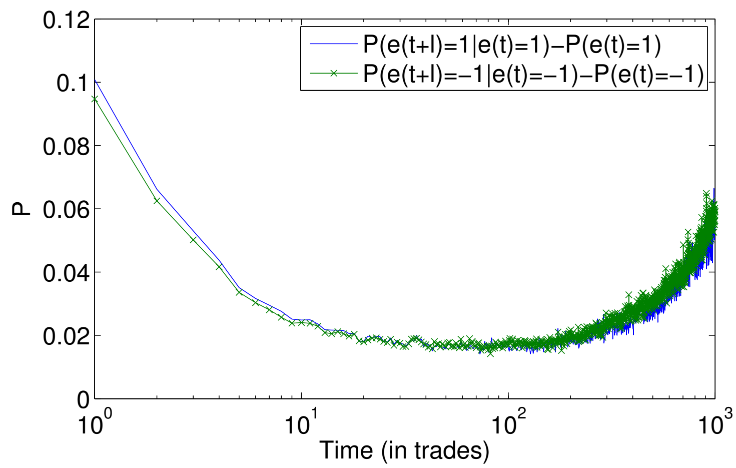

Figure 7. Conditional probability that a trade l trades later has the same sign

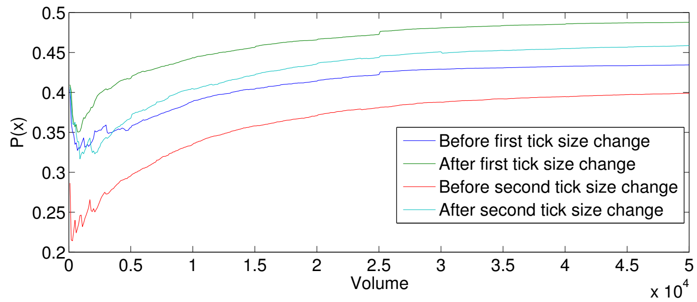

FIGURE 8. Probability that a transaction with volume less than x causes a price change for RTR in the two weeks before and after a tick size change

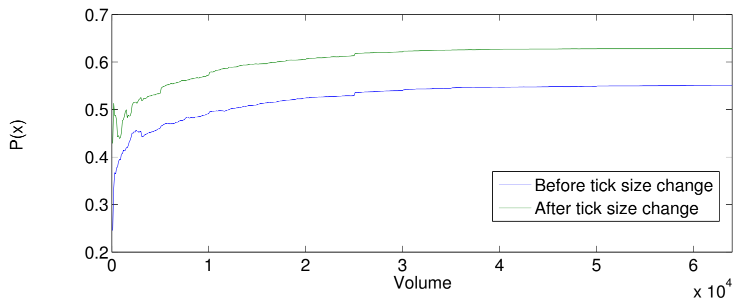

FIGURE 9. Probability that a transaction with volume less than x causes a price change for BSY in the two weeks before and after a tick size change

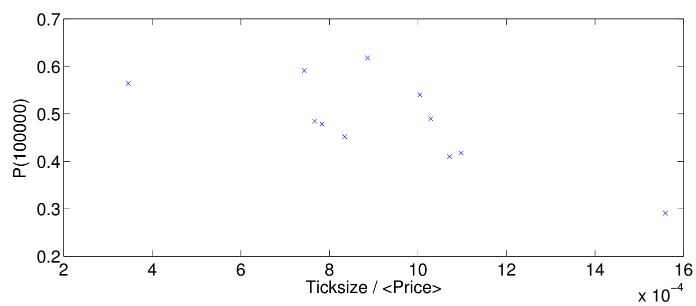

FIGURE 10. Probability that a transaction causes a change in price plotted against the tick size divided by the average price. Note that each point represents a stock, although RTR has 3 points (since it has 3 different tick sizes) and BSY has 2. Correlation coefficient of -.74

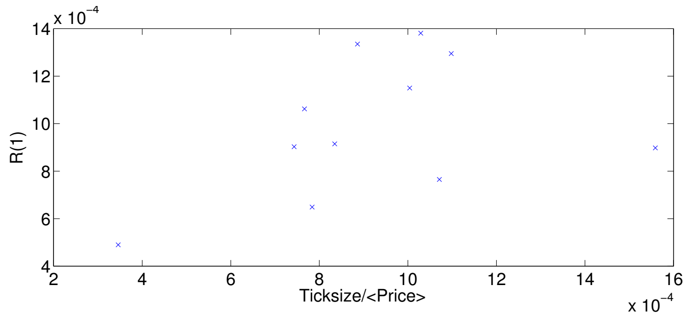

FIGURE 11. Value of $R(1)$ conditioned on a price change plotted against the tick size divided by the average price. Correlation coefficient of .41