A Model for Revolution Based on Multiplex Networks

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Abstract

A multiplex network is an ideal representation for modeling agents that interact in different ways. The present paper develops a dynamic agent-based study of revolutions under authoritarian regimes. We argue that formal and informal social interactions influence the political stance of the population in very different ways, particularly in contexts in which not all political opinions can be freely expressed. To account for this, society is modeled using a multiplex network with two layers: the formal network and the informal one. The formal network is a simple balanced tree that represents the set of professional interactions, with the dictator at the root. The informal network is a Barabasi-Albert network that represents friendship relationships. Every period some agents interact in one of the two networks. After interacting, depending on his position in the formal network and the information he receives, an agent decides whether to support or oppose the dictator. We study the fraction of agents who oppose the dictator as a function of two parameters: the fraction of agents that engage in social interaction p in each time step and the sensitivity of agents to their position in the formal network α. We find a sharp phase transition as p increases above zero, illustrating the importance of social interactions in facilitating revolutions.

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I. INTRODUCTION

Communication across society is crucial for opinion building, and at the same time it facilitates individuals to create an expectation of the proportion of the population that supports different views. This is particularly true in countries without political freedom, in which political opinions opposed to the established power are frequently prosecuted and punished. In this paper we study how people's political stance is influenced by social interactions in a society governed by an authoritarian regime.

In our model society is affected by two competing dynamics, with agents weighting conflicting interests against each other in the two layers. On one hand, they are part of a professional network which is hierarchical. In this network political information moves only downstream, from higher positions to lower ones. Since agents in a higher professional position tend to be favored by the status quo, they will be more likely to support the current regime. When an agent talks about politics in the formal network he will try to convince his subordinates to support his political stance. In the informal network however, agents talk only to their friends, revealing their political allegiance honestly and also getting to know their friends'. Knowing friends' views has a direct effect on the opinion of an agent. There are several arguments to justify why this might be the case. Behavioral effects such as the tendency to conform, with agents disliking to have different opinions than those they perceive among their friends, might be an important element to take into account. Other issues that might explain the tendency to follow the majority are the existence of social norms and peer pressure, with the potential punishment they imply for those diverging. Another argument particularly significant is that in a context with political tensions, in order to predict the probability of success of different political options people will need to estimate the prevalent view from their communication with others, frequently over-weighting the opinions of those who are socially closer to them. The evolution of these dynamics over time will determine the evolution of political allegiances across society, with a revolution being triggered when enough people gets to oppose the current regime.

We use multiplex networks to model this process. Multiplex networks are structures that represent agents interacting with several types of connections. Each layer in the multiplex may have its own distinct dynamical process. Here we study a simple model in which the formal network is assumed to be a tree with a *dictator* at the root. The same agents also participate in the informal layer, a Barabasi-Albert network. Aside from the dictator who always supports itself, each agent can choose to support or oppose the dictator. We study the success of a revolution as a function of the frequency of social interaction and the sensitivity of agents to their position in the formal network. A revolution succeeds if all agent oppose the dictator.

We study the success of revolution as a function of the frequency of social interaction and the sensitivity of agents to their position in the formal network. We find even in a strongly authoritative regime, where opposition is very costly, small social interactions can cause the opposition to spread in the population facilitating a global revolution.

II. IMPACT OF SOCIAL MEDIA

The study of social media and its influence on political discourse has become a hot topic, particularly after the technologically mediated Arab Spring. Broadly speaking the Internet dramatically reduces the cost of organizing protests [1, 10]. However, Internet access does not automatically increases participation in protests [5].

Social media contributes to the organization of protests in two principle ways: *reinforcement* and *innovation* [7]. On the one hand the use of social media may reinforce the traditional collective action through transnationalization and mobilization [4, 11], on the other hand the it creates new social-media driven activism [2, 6].

Prevalence of the Internet and, in particular, of social networks fosters communities built on collective identities and facilitates political mobilization by creating the infrastructure to achieve a critical mass (Harlow 2013). In addition, in non-democracies, the Internet is expected to decrease the influence of established media organizations and lower the dependence of political agenda on institutional structures [3].

III. THE THEORETICAL MODEL

A. The Networks

Our model consists of N agents each of which participate in two network layers denoted by $m \in \{1, 2\}$. Each layer is represented by an adjacency matrix A^m containing L_m links. Agent j is a neighbor of agent i in layer m if $A^m_{ij} = 1$ and is not if $A^m_{ij} = 0$.

Every agent is part of a professional (or formal) network A^1 and in a friendship (or informal) network A^2. These networks are depicted in Figure 1. The friendship network A^2 is defined by an irregular network with small-world property. We define this network to be the Barabsi-Albert network. In this model the agents are added to the network one by one. Each new agent will be connected to two existing agents with a probability proportional to their degree. The formal network A^1 is a hierarchical network. We take this network to be a balanced tree in which the dictator occupies the root. Initially, the agents that are at the leaves oppose the dictator and the rest of agent support it. In our experiment, the branching factor of the tree is 3 and the tree has 6 layers. The agents at leaves are shown with color red on Figure 1a. You can see the same agents with color red on the informal network on Figure 1b. You can see the interactions of the agents in the informal network does not obey their position in the formal network. The dynamic game proceeds in discrete time, with periods $t = 0, 1, 2, 3...$ alternating between the two networks.

B. The game

The dynamics of our model is a result of the games played by agents on each multiplex layer at each discrete time step t. Each layer has its own distinct game. At time $t = 0$ the agents are initialized with their state and the game commences. In all subsequent odd time steps $t = 1, 3, 5, \ldots$, the agents interact and play on the informal network. In all the even time steps, the agents interact and play on the formal network. The game continues until a termination criteria is met.

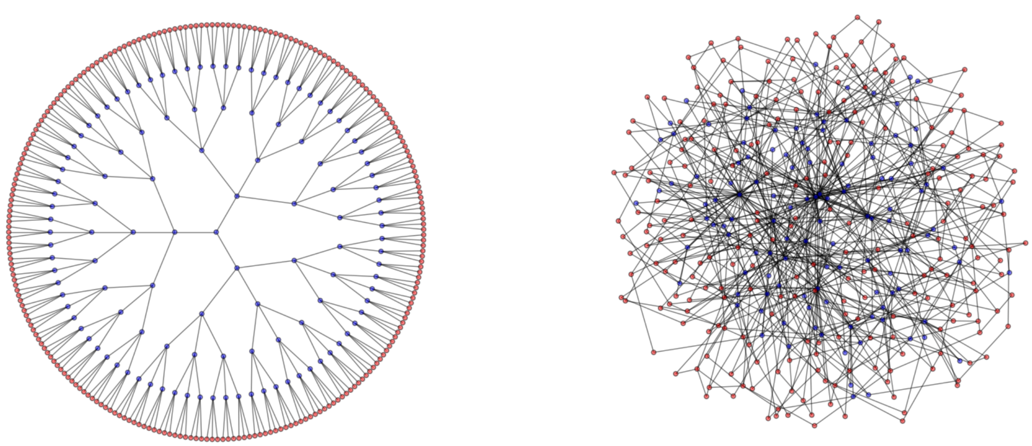
The initial configuration of the agents at $t = 0$ is set by assigning all the agents that at the leaves of the tree to "oppose" the dictator and all the other agents are set to "support" it. The game then proceeds on the informal network for odd t as follows. Each agent i is selected with independent and identical probability $\bar{p}$ to update its state. The agent then calculates k_i the fraction of its direct neighbors who oppose the dictator and updates its state w_i as follows:

$$w_i = \begin{cases} \text{``oppose''}, & k_i \geq k^* = \frac{1}{d_i^\alpha} \\ \text{``support''}, & \text{otherwise.} \end{cases} \tag{1}$$

Here, d_i is the shortest-path length from the agent i to the dictator and the parameter α controls the sensitivity of agents decision to its distance from the dictator. For $\alpha = 0$, all the agent's neighbors have to oppose the dictator for the agent i to oppose the dictator as well, independent of the agent's position in the formal network d_i. For $\alpha > 0$, the k^* decreases as d_i increases and the larger the α the sharper the decrease will be.

In each even time step t the agents play on the formal layer as follows. A single agent is selected randomly and it will update the state of its subordinates to its own state.

The system's dynamics alternates between the games on each layer until a termination criteria is met. In our experiment, we run the game until $t = 3000$. We say a revolution has



(a) Formal network A^1 (b) Informal network A^2

FIG. 1. The two layers of the multiplex network. Fig (a) shows the structure of the formal network defined by the matrix A^1. Fig (b) shows the informal network given by the matrix A^2. The network is generated with the Barabasi-Albert model. All agents are present in both networks.

taken place if all the agents except for the agent at the root of the formal network oppose the dictator.

IV. THE EXPERIMENTAL SET UP

A. Initial Configuration

The position of the agents in the professional network determines the payoff agents get in each round played in this layer. For each agent i the payoff function is defined as $\pi(d_i)$, where d_i refers to the distance[8] between i and the dictator. $\pi(d_i)$ is decreasing on d_i, so those agents closer to the dictator in the professional layer receive a larger payoff than those who are farther away from him. A simple example of such a payoff function is:

$$\pi(d_i) = \frac{1}{d_i + 1} \tag{2}$$

so the dictator would get $\pi(0) = 1$ and an agent at distance 5 from him $\pi(5) = 1/6$.

The position of a given agent in the professional network will determine his initial state as regarding his support or opposition to the regime. There are several possible ways of mapping payoffs into states. Some viable mappings are:

- Define a function $p(\pi)$, such that a given payoff is associated with a certain initial probability of liking the regime, with p increasing on π.
- Define a criteria, by which all the agents with a payoff higher than π^* like the regime, while all those with a lower payoff do not. This could be, for example, the case in which all those who under the current regime get a payoff higher than what they would get after a revolution which divides the remaining payoff evenly support the regime, and those who currently get less than they would if the whole income were split evenly or not. Defining total after-revolution payoff as $(\Pi - \delta)/N$ -where Π is the total payoff in the professional network and δ is the reduction in total payoff caused by an eventual revolution.
- Arbitrarily define a threshold distance after which agents do not like the current regime. For example assume that all the agents in the lowest hierarchy do not like the regime, while any other does.

Notice that the payoff in this network is independent of an agent state (which they might not even reveal). Once initial states are determined, period 0 ends and the game moves to the friendship network A^2.

B. The game dynamics

In the friendship network agents talk only to their friends[9] and if two friends interact in a given period t, they are always honest about their state. The payoff agents receive from this social interactions is higher if they conform with their friends' state, but players' optimal choice is determined by their payoff in the professional network.

In every even period, each player in A^1 is chosen with probability $\bar{l}$. The chosen players talk to their subordinates -those first neighbors in the lower hierarchy- and (if some of these have a different state than his) convinces them of switching to his own state with probability s.

C. The end of the game

The game finishes either when all the agents like the regime or when all the agents (except the dictator, which never switches) dislike the regime. Alternatively we can establish a threshold X^* such that the game finishes if either the amount of players who like the dictator or the amount of players who dislike the dictator drops below this level.

D. Network topologies and the Multiplex

In the current setup we considered a multiplex with two network layers: a formal network represented as a tree and an informal network represented as the Barabasi-Albert network. An obvious extension of this work would include the study of the effect of various parameters and network models as the formal and informal networks. Another important extension would be to include more than two layers each with its own game. Such a generalization will be valuable to gain insight into the nature of revolution and other social and political mobilization in complex social structures.

V. RESULTS

Figure 2 shows the result of five runs of the game with identical parameter values $\alpha = 0.45$ and $p = 0.2$. The horizontal axis is the time and the vertical axis shows the fraction of agents who support the dictator as the game progresses. There is an initial spike in the number of supporters suggesting that initially the supporters at one level before the leaves are able to persuade their subordinate to follow their preference. However, the trend stops as a few agents in higher ranks of the formal network turn against the dictator. This starts a stable and gradual decrease in the number of supporters until a catastrophic phase transition occurs where a large number of agents turn against the dictator. This presumably occurs when very high ranking agents in the formal network have enough interactions with opposite state that convince them turn their state. This will cause a cascade since their persuade their subordinate to comply with their decision and therefore a large number of agents turn their state quickly causing a rapid drop in the number of supporters.

The rate of gradual decrease in the number of supporters and the time when the phase transition occurs depends on supporter threshold levels. The transition is heavily influenced by the characteristics of the underlying informal network. As a result it is obvious that we expect a large variation in these measures.

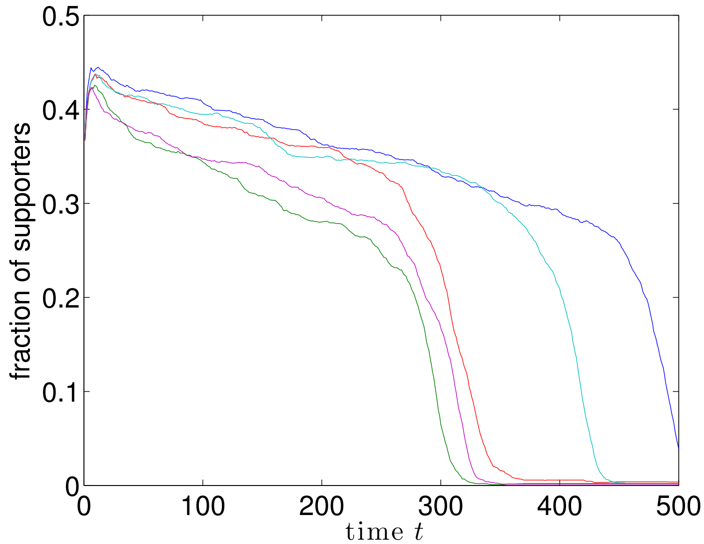


FIG. 2. Fraction of supporters of the dictator for five runs of the game. The parameters are $p = 0.2$ and $\alpha = 0.45$.

Next, to understand the sensitivity of this result to the variation in the parameters we measure the time to revolution, i.e., the amount of time it takes for the number of supporters to go to zero. We plot this measure as a function of p and α. Figure 3 shows the result of study. We systematically very the parameters between 0 and 1 with 0.05 increments. For each parameter combination the experiment is run 20 times and the results are averaged. To enhance the visualization, we used a polynomial fit to the data points. The results are very revealing. For $\alpha < 0.4$ the revolution never occurs independent of p. Also for $p > 0.2$ the results only depend on α and not p. In this regime for $\alpha > 0.6$ the revolution happens immediately after the game starts suggesting that the dynamics in the informal network dominates the game and for $\alpha < 0.4$ the revolution never occurs meaning that the formal network dominates the game. For $p < 0.2$ the revolution can still happen but it takes a long time. However, it is interesting to see that even at $p = 0.2$, which means with only 20% of the population participating in informal interaction we observe the revolution. This speaks to the effectiveness of social media in facilitating and accelerating revolutions.

VI. EXTENSIONS AND FUTURE WORK

The results presented in this work is from a specialized model that was developed to show dynamics between formal and informal networks. The full theoretical model includes many generalized features study of which we defer to future extensions of this work. Here we briefly mention some of the major generalizations.

The first generalization of this work is to study the effects of the network structure of formal and informal networks on the findings of the current report. Specifically, how does our result depend will change if the branching factor in the formal network changes or if we apply heterogeneous branching factors? Also, how does the result change if we introduce

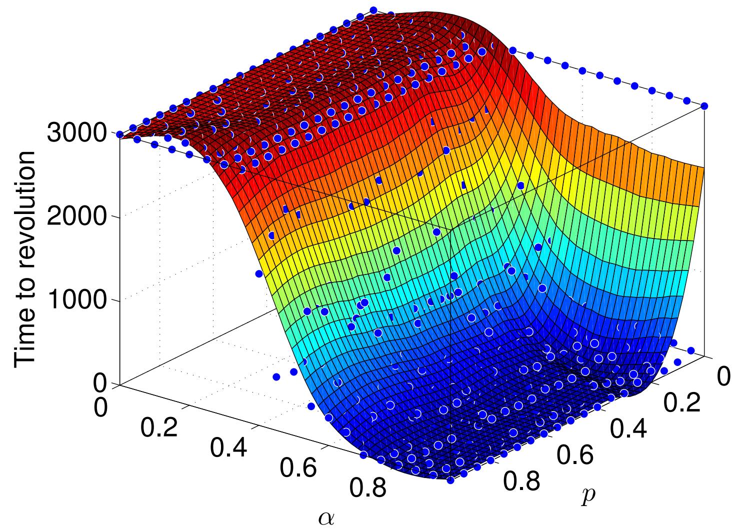


FIG. 3. The time it take for revolution to succeed as a function of α and p For $p > 0.2$ this measure is independent of p, where for $\alpha < 0.4$ the revolution never takes place and for $\alpha > 0.6$ it happens immediately after the game starts. Even with only a small fraction of agents with social interaction the revolution takes place instantaneously in large α regime.

irregularities in the formal network.

The second extension is generalization to play other types of games or varying parameters of the games. This also includes the study of games on more than two layers.

An important missing analysis is a mathematical description of the current result to provide us with theoretical insights into the nature of the observed behavior. In the future we will work on a generalized mathematical framework to help us analyze our results in more theoretical setup.

VII. CONCLUDING REMARKS

We used a multiplex network structure to model revolutions in authoritarian systems. The multiplex structure allows us to model complex collective dynamics that are results of distinct competing or cooperating processes that are taking place in a single population. An example of such a complex scenario is individuals in a society that engage in many different formal and informal interactions with each other, which shape their behavior. We saw that the mix of multiplex networks, game theory and agent based modeling gives us a powerful tool to gain insight into the emergence of collective social phenomena such as revolutions.

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[8] Number of edges in the shortest path.

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